

Book of Abstracts of Plenary Talks XXV Brazilian Algebra Meeting

State University Campinas, December 3 - 7, 2018

	December 3rd	December 4th	December 5th	December 6th	December 7th
8h00 - 8h50:	Mini-courses 2,4,7	Mini-courses 2,4,7	Mini-courses 2,4,7	Mini-courses 2,4,7	Mini-courses 2,4,7
9h00 - 9h50:	Mini-courses 1,3,5,6	Mini-courses 1,3,5,6	Mini-courses 1,3,5,6	Mini-courses 1,3,5,6	Mini-courses 1,3,5,6
9h50 - 10h20:	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10h30 - 11h20:	Ulrich	Vainsencher	Aljadeff	Pellikaan	Marques
11h30 - 12h20:	Sidki	Hernandez	Panario	Lanzilotta	Korchmaros
12h20 - 14h00:	Lunch	Lunch	Lunch	Lunch	Lunch
14h00 - 15h00:	Sessions	Sessions	Sessions	Chari	Sessions
15h00 - 16h00:	Sessions	Sessions	Sessions	Posters	Sessions
16h00 - 16h30:	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
16h30 - 18h00:	Sessions	Sessions	Sessions	Posters	Sessions

Group Gradings on Algebras

Yuri Bahturin

I would like to talk on some new developments in the theory of group gradings on algebras. The area is quickly growing, embracing non-algebraically closed fields, wider classes of non-associative algebra, etc. I would also like to talk about some new achievements in the theory of graded polynomial identities and graded representations of Lie algebras.

Introdução à Geometria Projetiva

Carolina Araujo (Instituto de Matemática Pura e Aplicada - Brazil)

1a aula. Origem da Geometria Projetiva: a teoria da perspectiva e a pintura renascentista. Pontos no infinito. O plano projetivo.

2a aula. Retas no plano projetivo. Transformações projetivas. A razão cruzada. Aplicações.

3a aula. Coordenadas homogêneas. Curvas projetivas. Classificações das cônicas.

4a aula. A estrutura de grupo em uma cúbica não-singular. Aplicações em criptografia.

5a aula. Teorema de Bézout. Dualidade e polaridade.

Pré-requisitos: Álgebra linear, noções básicas de grupos e fatoração em anéis de polinômios.

The Maximal Subgroups of the Symmetric Group

Martino Garonzi (Universidade de Brasília - Brazil)

Topics: Finite permutation groups, transitive and primitive actions, minimal normal subgroups and socle of a finite group, structure of the socle of a primitive group, O’Nan-Scott theorem for finite primitive permutation groups: affine groups, almost-simple groups, product actions and diagonal actions.

Abstract:

Let S_n and A_n denote respectively the symmetric and alternating group on n letters. In this minicourse we will present the modern classification of the maximal subgroups of S_n and A_n and their indeces, via the O’Nan-Scott theorem, the classification of finite primitive groups.

A group G acting faithfully and transitively on $\{1, \dots, n\}$ is said to be “primitive” if the point stabilizers are maximal subgroups of G . Apart from A_n (which is maximal in S_n) the maximal subgroups of S_n (and A_n) are essentially of two types: the first type is “what you would expect” (stabilizers of natural actions), the second type is given by primitive maximal subgroups, which represent a “local obstruction”. As a corollary of the O’Nan-Scott theorem, the set of numbers n such that S_n (and A_n) present no such local obstruction has density 1.

The discussion will allow us to show how primitive groups can be used to understand finite groups in general. We will also present some recent applications of the classification.

References

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O que é um loop?

Maria de Lourdes Merlini Giuliani (Universidade Federal do ABC - Brazil)

Dylene Agda Souza de Barros (Universidade Federal de Uberlândia - Brazil)

Rodrigo Lucas Rodrigues (Universidade Federal do Ceará - Brazil)

Rosemary Miguel Pires (Universidade Federal Fluminense - Brazil)

Mariana Garabini Cornelissen Hoyos (Universidade Federal de São João Del Rei - Brazil)

Giliard Souza dos Anjos (Universidade Federal do ABC - Brazil)

A Teoria de Loops se insere na linha de pesquisa de álgebras não associativas. Enquanto a teoria de grupos em geral está madura, a teoria de loops é relativamente jovem, tendo surgido há menos que um século. Alguns resultados de investigações realizadas nas últimas décadas revelaram que quasigrupos e loops possuem um importante papel em diversas especialidades da subálgebra de álgebra, a exemplo de análise combinatória, teoria de códigos, geometria (projetiva, diferencial, hiperbólica, esférica, web), teoria de grupo (finita e Lie) e teoria de nós.

Este minicurso se destina a participantes com conhecimento básico em teoria de grupos que tenham interesse em estender seus conhecimentos para o campo de loops. É esperado um amplo interesse em quasigrupos e loops, uma vez que estes prestam ao espectro completo de técnicas da álgebra universal.

Aula 1: Abordagem histórica. Primeiras definições. Quasigrupo. Loop. Subloop. Subloops relevantes e suas similaridades com grupos. Exemplos. Loop dos Octônios.

Aula 2: Loops de Moufang. Identidades relevantes. Núcleo de um loop de Moufang. Teorema de Moufang. Loops de Chein e sua similaridade com grupos diedrais. Teorema de Lagrange.

Aula 3: Aplicações internas. Homomorfismos de loops, subloops normais e subloop quociente. Propriedades inversas de loops. Caracterização de subloops. Relações entre os núcleos à esquerda, à direita e do meio.

Aula 4: Álgebras alternativas. Loop dos elementos invertíveis de uma álgebra alternativa. Álgebra de Zorn e o loop projetivo linear.

Aula 5: Loops automórficos e suas propriedades. Loops automórficos de Lie. Problemas de pesquisa em aberto.

Curvas Maximais

Paulo César Oliveira (Universidade Regional do Cariri - Brazil)

Fernando Torres (Universidade Estadual de Campinas - Brazil)

Palavras chaves: corpos finitos, curvas sobre corpos finitos, Teoria de Stöhr-Voloch, cota de Hasse-Weil, curva Hermitiana, curva GK.

Consideramos curvas \mathcal{X} (projetivas, não singulares, geometricamente irredutíveis) definidas sobre um corpo finito \mathbf{F} de ordem ℓ . Estamos interessados no número de pontos \mathbf{F} -racionais $N = \#\mathcal{X}(\mathbf{F})$, que pelo teorema de Hasse-Weil satisfaz a desigualdade $|N - (\ell + 1)| \leq 2\sqrt{\ell} \cdot g(*)$, onde $g = g(\mathcal{X})$ é o gênero de \mathcal{X} . Neste curso estamos interessados nas curvas que atingem a cota superior em (*), i.e., $N = \ell + 1 + 2\sqrt{\ell}g$ sendo $\ell = q^2$ um quadrado. Estas curvas serão ditas *\mathbf{F} -maximais* [2], [5, Ch. 10].

I) Consideramos o conjunto $\mathbf{M}(q^2) := \{g \in \mathbb{N}_0 : \text{Existe curva } \mathbf{F}\text{-maximal de gênero } g\}$ e discutimos a inclusão $\mathbf{M}(q^2) \subseteq [0, g_2] \subseteq \{g_1\} \subseteq \{g_0\}$, onde $g_0 := q(q-1)/2$ (cota de Ihara), $g_1 := \lfloor (q-1)^2/4 \rfloor$, $g_2 := \lfloor (q^2 - q + 4)/6 \rfloor$. Para isto estudamos a geometria de um sistema linear (muito amplo e intrinsecamente) associado a uma curva maximal, via a teoria de Stöhr-Voloch, cota de Castelnuovo, cota de Halphen [6].

II) Para $g \in \mathbf{M}(q^2)$ estudamos a unicidade da curva \mathcal{X} com $g(\mathcal{X}) = g$, em especial para $g = g_0, g_1, g_2$. De fato $g(\mathcal{X}) = g_0$ se, e somente se, \mathcal{X} admite um modelo plano do tipo $y^{q+1} = x^q + x$ (Curva Hermitiana sobre \mathbf{F})

III) Consideramos uma forma de gerar curvas maximais (atribuído a Kleiman/Tate/J.P. Serre); em particular, curvas \mathbf{F} -dominadas pela curva Hermitiana são \mathbf{F} -maximais. Nem toda curva \mathbf{F} -maximal admite esta dominação : temos a curva GK (Giulietti-Korchmáros [3]) e certos recobrimentos [7], [4]. Nestes exemplos $q = t^a > 8$, $a \equiv 0 \pmod{3}$; discutiremos a possibilidade de obter exemplos de tipo GK para $a \not\equiv 0 \pmod{3}$ [1], e portanto definir a onipresença das curvas Hermitiana e GK no contexto das curvas maximais.

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Invariant theory of Ore extensions

João Schwarz (Universidade de São Paulo - Brazil)

In this mini-course we aim to an overview of results of invariant theory of the polynomial algebra which extends to Ore extensions. We begin with a survey of results from classical invariant theory of the polynomial algebra: (1) Hilbert-Noether Theorem, (2) Chevalley-Shephard-Todd Theorem, (3) Jung-van der Kulk Theorem; and some more specific topics such as (4) Miyata's Theorem and (5) Noether's Problem. Next we introduce Ore extensions; our main examples will be rings of differential operators and certain quantum algebras. We discuss generalizations of the above results to noncommutative setting: (1)' Montgomery-Small Theorem, (2)' the work of Alev, Polo, Kirkman, Kuzmanovich and Zhang on Galois embeddings of noncommutative algebras, (3)' the structure of the automorphism group of many Ore extensions of (Gelfand-Kirillov) dimension 2, (4)' Miyata's Theorem for Ore extensions and (5)' noncommutative versions of Noether's Problem. We shall discuss many aspects of the later and applications to the Gelfand-Kirillov Conjecture. Finally, we introduce a certain nice category of modules of certain invariants of Ore extensions: Gelfand-Tsetlin categories.

1 Bibliography

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The Specht Problem and Kemer's Theorems

Irina Sviridova (Universidade de Brasília - Brazil)

The principal propose of this mini-course is to represent a proof of classical theorems of A.R.Kemer dedicated to a positive solution of Specht problem.

Observe that the problem of Specht is one of the most important question in the theory of polynomial identities. The initial question was stated by W.Specht in 1950 [36] for associative algebras over a field: "Is it true that any associative algebra over a field has a finite set of identities that implies all identities of this algebra?" This set of identities is called a base of identities of the algebra. In general, a question of existence of a finite base of identities is the central question for identities of any algebraic system (e.g., groups, semigroups, rings, associative and non-associative algebras, e.g., Lie algebras, Jordan algebras, super-algebras, graded algebras, etc.). Really, this is the question of existence of an effective description of identities of these systems. The resolution of this question is of the great importance.

A positive solution of the classical problem of Specht for associative algebras over a field of characteristic zero was given by A.R.Kemer [25], [26]-[29]. The theorems of Kemer are fundamental for the theory of identities of associative algebras and have a great potential for applications. Because they provide a possibility to build a nice algebra-carrier for the given ideal of identities in an effective manner. The theorems claim that any T-ideal (an ideal closed under all endomorphisms) of the free associative algebra over a field of characteristic zero is an ideal of identities of some algebra with a good structure, which can be effectively described using the classical results of the ring theory. Besides that the structure constants of this algebra (e.g., such as the dimension of the semisimple part, the degree of nilpotency of the Jacobson radical, etc.) are strongly related to the numerical parameters of identities. This is a powerful instrument for studying identities of algebras and solving other problems related to identities. The importance of the results of A.R.Kemer is also confirmed by the existence of several more recent generalizations and modifications of them for other algebraic systems.

We will consider the classical case of associative algebras over a field of characteristic zero, and present a proof of the results of A.R.Kemer. We are going to prove that the ideal of super-identities of a finitely generated PI super-algebra coincides with the ideal of super-identities of some finite-dimensional super-algebra. We also will see that the ideal of polynomial identities of any PI-algebra (not necessarily finitely generated) coincides with the ideal of the identities of the Grassmann envelope of a finite-dimensional super-algebra. In particular, this implies that the ideal of identities of any algebra is finitely generated as a T-ideal, which is the positive answer to Specht problem.

Besides that, we will study, how the structure parameters of algebras-carriers reflect on their identities. Notice, this is also one of the basic questions - to study, how the properties of algebras can be expressed by their identities.

Let us mention that although the results are classical, fundamental and very important to the theory of identities, all of the existing demonstrations (including the original Kemer

proof) are quite big and rather difficult to study. We will try to present the arguments in a more complete and detailed manner and at the same time more clearly and as simple as possible. The prerequisites of the mini-course are knowledge of the classical results of the ring theory (Wedderburn-Malcev theorem, Jacobson radical properties of finite-dimensional algebras, etc.), and of the basic notions of the theory of identities (a T-ideal, multilinearization process, partial linearizations, alternating polynomials, etc.) (see, e.g., [17], [18], [19]).

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