Book of Abstracts
XXV Brazilian Algebra Meeting

State University Campinas, December 3 - 7, 2018

Session: Group Theory

Room 151 IMECC

	December 3rd		**December 4th**		**December 5th**
14h00 - 14h40:	Shusterman	14h00 - 14h40:	Del Río	14h00 - 14h40:	Kochloukova
14h45 - 15h05:	Amato	14h45 - 15h25:	Schneider	14h45 - 15h25:	Tanushevski
15h10 - 15h30:	Lima				
15h35 - 15h55:	Bastos	15h30 - 15h50:	Rocha	15h30 - 15h50:	Pinto
16h00 - 16h30:	Coffee Break	16h00 - 16h30:	Coffee Break	16h00 - 16h30:	Coffee Break
16h30 - 16h50:	Dantas	16h30 - 16h50:	Hussain	16h30 - 16h50:	Almeida
16h55 - 17h15:	Contreras	16h55 - 17h15:	Giuliani	16h55 - 17h15:	Mendonça
17h20 - 17h40:	MacQuarrie			17h20 - 17h40:	Tosti

A conjecture on the BNS-invariant for Artin Groups

Kisnney Emiliano de Almeida (Universidade Estadual de Feira de Santana - Brazil)

Abstract:
The Bieri-Neumann-Strebel-invariant, or Σ^1-invariant, is a geometric invariant of finitely generated groups that can be used to decide if a given subgroup above the commutator is also finitely generated. It is the first of a series of topological and homological Σ-invariants that can be similarly used to study finiteness properties of groups. Artin groups form a large class of groups, combinatorially constructed from underlying graphs, source of interesting examples in geometry and group theory. Meier, Meinert and Van-Wyk have obtained a partial description of Σ^1 for Artin groups - given an Artin group G they establish a necessary and a sufficient condition, depending on the underlying graph, for a character of G to be on the invariant. We conjecture, as stated in a joint work with Kochloukova, the above sufficient condition to be also necessary, resulting on a complete description for all Artin groups. This conjecture has been proven for some classes of Artin groups and no counterexample has been found. We will talk about these ideas and results.

Highly arc transitive descendant-homogeneous digraphs with prime out-valency

Daniela Amorim Amato (Universidade de Brasília - Brazil)

Abstract:
The *descendant set* of a vertex in a digraph D is the induced subdigraph on the set of vertices reachable from the given vertex by an outward-directed path. Descendant sets in highly arc transitive digraphs having the additional transitivity property of *descendant-homogeneity* have considered previously. A digraph D is descendant-homogeneous if it is vertex transitive and any isomorphism between finitely generated subdigraphs of D extends to an automorphism of D, where a subset of vertices of D is finitely generated if it is a union of finitely many descendant sets.

We consider highly arc transitive digraphs D of finite out-valency with an extra condition on the descendant set. We show that if L is an infinite line in D and F is the subdigraph induced on the descendants $\mathrm{desc}(L)$ of L, then F is a 2-ended digraph with equal in- and out-valencies that has property Z. In particular, when D is descendant-homogeneous and has prime out-valency p, then F is isomorphic to the digraph Δ_p. As a consequence, we get a complete classification of the highly arc transitive descendant-homogeneous digraphs with prime out-valency which have property Z.

Engel conditions in groups

Raimundo de Araújo Bastos Júnior (Universidade de Brasília - Brazil)

Abstract:
We describe local nilpotency criteria arising from Engel contitions.

On state-closed representation and virtual endomorphisms

Alex Carrazedo Dantas (Universidade de Brasília - Brazil)

Abstract:
In this work, we study constructions of state-closed groups not necessarily transitive by means of a definition of representation in terms of virtual endomorphisms. We show that a group is isomorphic to a state-closed group if and only if such a representation exists and is faithful. Furthermore, we show that a free abelian group of infinity rank and the Lamplighter group $\mathbb{Z} \wr \mathbb{Z}$ are state-closed of degree 3 and finite-state.

Work in join with Tulio Santos and Said Najati Sidki.

Length of Finite Groups

Yerko Contreras Rojas (Universidade Federal do Sul e Sudeste do Pará - Brazil)

Abstract:
Let p be a prime. Every finite group G has a normal series each of whose quotients either is p-soluble or is a direct product of nonabelian simple groups of orders divisible by p. The non-p-soluble length $\lambda_p(G)$ is defined as the number of non-p-soluble quotients in a shortest series of this kind.

We deal with the question whether, for a given prime p and a given proper group variety $\mathfrak{V}$, the non-p-soluble length $\lambda_p(G)$ of a finite group G whose Sylow p-subgroups belong to $\mathfrak{V}$ is bounded.

In joint work with Pavel Shumyatsky, we answer the question in the affirmative in some cases (working separately the case $p = 2$) for varieties of groups in which the commutators have conditions that depends on exponent conditions and Engel conditions.

On the group of half automorphisms of the Cayley-Dickson loop

Maria de Lourdes Merlini Giuliani (Universidade Federal do ABC - Brazil)

Abstract:
A half-isomorphism $\varphi : G \longrightarrow K$ between multiplicative systems G and K is a bijection from G onto K such that $\varphi(ab) \in \{\varphi(a)\varphi(b), \varphi(b)\varphi(a)\}$ for any $a, b \in G$. It is well known that if G is a group then φ is either an isomorphism or an anti-isomorphism. Although this result carries over to Moufang loops of odd order, it does not hold for all Moufang loops that are of even order. If $K = G$ then φ is called half-automorphism. We say that the half-automorphism φ is trivial if it is either an automorphism or an anti-automorphism.

The Cayley-Dickson loop C_n is the multiplicative closure of basic elements of the algebra constructed by n applications of the Cayley-Dickson doubling process. Cayley-Dickson loops are diassociative 2-loops, but also Moufang loops. An example of a nontrivial half-automorphism was firstly identified in an octonion loop C_4 of order 16. In my talk I will show that every Cayley-Dickson loop has nontrivial half-automorphism and present the group of half-automorphisms of C_4.

These results were obtained in collaboration with Liudmila Sabinina and Peter Plaumman.

Random walks on relatively hyperbolic groups
The invariant ring of $\mathrm{Aut}(V, H)$

Fawad Hussain (Abbottabad University of Science and Technology - Pakistan)

Abstract:
In this talk we compute the invariant ring of the subgroup of $\mathrm{GL}(V)$ given below

$$\mathrm{Aut}(V,H) = \{l \in \mathrm{GL}(V) : H(lw_1, lw_2) = H(w_1, w_2) \forall w_1, w_2 \in V\}$$

where V is a vector space on F_{q^2} and H is a hermitian form which is singular on V.

Isoperimetric functions for metabelian groups

Dessislava Hristova Kochloukova (Universidade Estadual de Campinas - Brazil)

Abstract:
We will show that every finitely generated metabelian group embeds in a metabelian group of type FPm that has polynomial isoperimetric function. This is a joint work with M. Bridson, M. Kassabov and F. Matucci.

A generalization of Weiss' Theorem

John William MacQuarrie (Universidade Federal de Minas Gerais - Brazil)

Abstract:
Let G be a finite p-group with normal subgroup N. Weiss' Theorem states that a finitely generated lattice U for G over the p-adic integers is a permutation lattice, whenever the N-fixed points of U form a permutation lattice and the restriction of U to N is free. Of course, a given lattice might be a permutation lattice even when the resticition to N is not free. However, simply demanding that the N-fixed points of U and U restricted to N be permutation lattices, we cannot affirm that U must also be. I'll present conditions on U, more general than those of Weiss' theorem, which allow us to conclude that U is a permutation module. Joint work with Pavel Zalesskii.

On the number of cyclic subgroups of a finite group

Igor dos Santos Lima (Universidade de Brasília - Brazil)

Abstract:
This is a joint work with Martino Garonzi (UnB) accepted for publication in the Bulletin of the Brazilian Mathematical Society, New Series (2018). Let G be a finite group and let $c(G)$ be the number of cyclic subgroups of G. We study the function $\alpha(G) = c(G)/|G|$ We explore its basic properties and we point out a connection with the probability of commutation. For many families $\mathcal{F}$ of groups we characterize the groups $G \in \mathcal{F}$ for which

$\alpha(G)$ is maximal and we classify the groups G for which $\alpha(G) > 3/4$. We also study the number of cyclic subgroups of a direct power of a given group deducing an asymptotic result and we characterize the equality $\alpha(G) = \alpha(G/N)$ when G/N is a symmetric group.

The Σ-invariants of wreath products

Luis Augusto de Mendonça (Universidade Estadual de Campinas - Brazil

Abstract:
We will discuss the Σ-invariants of permutational restricted wreath products of groups. We present a full description of the 1-dimensional invariant (the so called BNS-invariant) and partial results for the 2-dimensional homotopical invariant. Under a reasonable hypothesis these partial results turn into a full picture of such invariant.

Homological properties of centre-by-metabelian pro-p groups

Aline Gomes da Silva Pinto (Universidade de Brasília - Brazil)

Abstract:
In the general context of pro-p groups, the properties of being finitely generated and finitely presented can be translated as the homological properties FP_1 and FP_2, respectively. Highest homological finiteness property FP_m is a generalization of finite presentability in this context.

In this talk we will present results obtained jointly with D. Kochloukova for centre-by-metabelian pro-p groups that satisfies highest homological finiteness property FP_m.

Recent progress on torsion units of integral group rings

Ángel del Río Mateos (Universidad de Murcia - Spain)

Abstract:
Let G be a finite group and let $V(\mathbb{Z}G)$ denote the group of units of augmentation 1 in the integral group ring $\mathbb{Z}G$. In the 1960s Hans Zassenhaus conjectured that every torsion unit of $V(\mathbb{Z}G)$ is conjugate in $\mathbb{Q}G$ to an element of G. The Zassenhaus Conjecture has been verified for some large families of groups including nilpotent groups (Weiss 1991) and groups whose derived subgroup is either a p-group (Hertweck, 2006) or cyclic (Caicedo, Margolis and del Río, 2013). The conjecture has been also verified for some non-abelian simple groups. We will explain some recent results by Margolis, del Río and Serrano on the Zassenhaus Conjecture for some special linear groups and projective special linear groups.

One of the methods to prove the Zassenhaus Conjecture consists in proving that the partial augmentations of a torsion unit are all non-negative. This is a result of Marciniak, Ritter, Sehgal and Weiss. The partial augmentation of an group ring element $\sum_g a_g g$, are the sums $\sum_{g\in C} a_g$ for C a conjugacy class of G. The so called HeLP Method is a technique to prove the Zassenhaus Conjecture, based on some conditions that the partial augmentations of a torsion element of $V(\mathbb{Z}G)$ satisfy. This was introduced by Luthar and Passi and extended by Hertweck.

Margolis and del Río have shown some inequalities satisfied by the partial augmentations of a torsion element of $V(\mathbb{Z}G)$. They call them the Cliff-Weiss inequalities because they follow from a result of Cliff and Weiss. In some cases these inequalities are stronger than the conditions of the HeLP Method. Margolis and del Río have developed an algorithm, based on the Cliff-Weiss inequalities, to produce candidates to counterexamples to the Zassenhaus Conjecture. We will explain the ideas behind this algorithm. Eisele and Margolis have shown that some of the candidates produced by this algorithm are indeed counterexamples to the Zassenhaus Conjecture. These are the first counterexamples of the Zassenhaus Conjecture.

Balanced presentations for fundamental groups of curves over finite fields

Mark Shusterman (University of Wisconsin-Madison - USA)

Abstract:
We show that the algebraic fundamental group of a smooth projective curve over a finite field admits a finite topological presentation where the number of relations does not exceed the number of generators.

Finite p-groups with few characteristic subgroups and a class of simple anti-commutative algebras

Csaba Schneider (Universidade Federal de Minas Gerais - Brazil)

Abstract:
A typical finite p-group has lots of characteristic subgroups, and so those that have few are worthy of investigation. Finite p-groups with two characteristic subgroups are elementary abelian. A finite p-group with three characteristic subgroups has exponent either p or p^2. In this talk, I will show that there is a bijection between the class of finite p-groups of exponent p^2 with precisely three characteristic subgroups and a class of semisimple non-associative algebras over the field of p elements. The correspondence extends to subgroups and subalgebras. I will also outline the construction of several classes of simple algebras that give rise to interesting examples of such finite p-groups. The work presented in this talk is a collaboration with Stephen Glasby and Frederico Ribeiro.

The theory of group boundaries

Lucas Henrique Rocha de Souza (Universidade Federal de Minas Gerais - Brazil)

Abstract:
This talk is about compactifications of groups that extends continuously the left multiplication action to the whole space. However, such generality does not work well. So, it is presented the notion of perspectivity: a geometric property of the compactification, that generalizes convergence actions, where the theory becomes well behaved.

The conjugacy problem for a class of generalized Thompson groups

Slobodan Tanushevski (Universidade Federal Fluminense - Brazil)

Abstract:
For a given group G, there is a generalized Thompson group $\mathcal{F}(G)$ that combines G with Thompson's group F. In this talk, I will discuss the conjugacy problem for $\mathcal{F}(G)$. This is joint work with Francesco Matucci.

Decision Problems in Homeomorphism Groups

Altair Santos de Oliveira Tosti (Universidade Estadual de Campinas - Brazil)

Abstract:
Monod's group $H := H(\mathbb{R})$ introduced in (Monod, *Proc. Nat. Acad. Sci.* **110**(12), 2013, 4524–4527.) is a group of piecewise projective orientation-preserving homeomorphisms of $\mathbb{R} \cup \{\infty\}$ which stabilize infinity and is another counterexample of the von Neumann-Day conjecture. The group H can also be regarded as homeomorphisms of $\mathbb{R}$: an element f is in H if there are finitely many points $t_1, t_2, \ldots, t_n$ such that on each interval $[t_i, t_{i+1}]$

$$f\colon t \mapsto \frac{a_i t + b_i}{c_i t + d_i}, \text{ where } a_i d_i - c_i b_i = 1, \text{ for suitable } a_i, b_i, c_i, d_i \in \mathbb{R}$$

and $f\colon t \mapsto (a_0 t + b_0)/d_0$ on $(-\infty, t_1]$ and $f\colon t \mapsto (a_n t + b_n)/d_n$ on $[t_n, +\infty)$. Given a subring A of $\mathbb{R}$, the subgroup $H(A)$ of H consists of all elements which are piecewise in $PSL_2(A)$ with breakpoints in $\mathcal{P}_A$, the set of fixed points of hyperbolic elements of $PSL_2(A)$.

In this talk we discuss current progress on the study of the conjugacy problem and centralizers in H and its subgroups by generalizing techniques developed in (Kassabov and Matucci, *Groups, Geom. Dyn.* **6**(2),2012, 279–315) and (Burillo, Matucci, Ventura, *Israel J. Math.* **216**(1), 2016, 15–59).

Posters Group Theory

Thompson-like groups and Graph Diagram Groups

Miguel Alfredo Del Río Palma (Universidade Estadual de Campinas - Brazil)

Abstract:
We study two families related to the Thompson groups F, T and V. The first of these is the Rearrangement Group of Fractals, introduced by Belk and Forrest in (Belk and Forrest, Rearrangement groups of fractals, to appear in Trans. Amer. Math. Soc.). This is a family of groups that consist of homeomorphisms between self similar structures. We study Graph Diagram Groups, which provide a generalization of the Diagram Groups defined by Guba and Sapir in (Guba and Sapir, Diagram Groups, *Mem. Amer. Math.*, **130**(620):viii+117, 1997). The idea of this new family of groups is to consider graph rewriting systems instead of string rewriting systems to define the diagrams.

We will study the relation between these two families and take advantage of the diagram structure in Graph Diagram Groups to show certain properties on the elements in it.

Furthermore, we will study properties of Diagram Groups that remain true for Graph Diagram Groups. For example we study the conjugacy problem, product properties and the abelianization map on these groups.

Context Free Groups

Bianca Boeira Dornelas (Universidade Estadual de Campinas - Brazil)

Abstract:
Context free groups are closely related to the Chomsky Hierarchy, which classifies languages in four types (0 to 3): recursively enumerable, context sensitive, context free and regular ones. Studying these languages allows one to define corresponding types of groups, where some properties will be strongly associated with properties of the language. Therefore, we will approach essential concepts such as graphs, languages and automata, in order to present these languages and then see in more detail properties of the type 2 one, which defines context free groups. We will see how some properties on these groups are related to properties of the language and some examples of context free groups.

Profinite genus of fundamental groups of Sol and Nil 3-manifolds

Genildo de Jesus Nery (Universidade de Brasília - Brazil)

Abstract:
We prove that the profinite genus of the fundamental group $\pi_1(M) \cong (\mathbb{Z} \times \mathbb{Z}) \rtimes_A \mathbb{Z}$ of a Sol 3-manifold M is equal to the order of the class group of the field $\mathbb{Q}(\lambda)$, where λ is an eigenvalue of the matrix A in $\mathrm{GL}_2(\mathbb{Z})$.

Automata generating free products of groups of order 2

Marcelo Sousa de Miranda Júnior (Universidade Estadual de Campinas - Brazil)

Abstract:
An automaton (or a Mealy automaton) A consists of a tuple (Q, X, π, λ), in which Q is a set of states, X is a finite alphabet, $\pi : Q \times X \to Q$ is a transition function and $\lambda : Q \times X \to X$ is an output function. Among all the types of automata, we highlight finite invertible automaton in order to define an automata group. A special family of automata which has fundamental importance to our work is the Bellaterra automata family, first studied during the summer school in automata groups at the University of Barcelona in Bellaterra, in 2004 (this is why these automata receive this name); such automata are defined by wreath recursion. On this presentation, we construct a family of automata (the Bellaterra automata) with $n \geq 4$ states acting on a rooted binary tree generating free products of cyclic groups of order 2 by going through the concept of automata group. This study is based on an article of Savchuk and Vorobets.

Elements of the alternating group as products of conjugate cycles and commutators

Ian Simões Ornelas (Universidade Federal de Minas Gerais - Brazil)

Abstract:
We give an overview of developments in identities relating to the alternating group. In 1951 it was shown that every element in the alternating group A_n can be written as a commutator of elements in S_n, in the same year it was shown that it could be written as a commutator of elements in A_n, various related theorems were proven in the intermediate years, leading to the central result which shows that every element in A_n can be written as a product of two cycles of length ℓ, for all ℓ between $\frac{3n}{4}$ and n.

Normal subgroups in limit group of Prime index

Jhoel Estebany Sandoval Gutierrez (Universidade Federal de Mato Grosso do Sul - Brazil)

Abstract:
Recently, D.H. Kochloukova and P.A. Zalesskii introduced and studied a class of pro-p groups which might be considered as the pro-p analogue of the class of limit groups and called it the class of *pro-p limit groups*. Motivated by one of their main results on pro-p limit groups they raised the following question.

Question. Let G be a non-abelian limit group, and let U be a normal subgroup of G of prime index p. Does this imply that $d(U) > d(G)$?

Here $d(G)$ denotes the minimum number of generators of a finitely generated group G. There are two homological invariants of a finitely generated group G which can be seen as a homological approximation of $d(G)$: The *rational rank* of G given by

$$\text{rank}_{\mathbb{Q}}(G) = \dim_{\mathbb{Q}}(G^{ab} \otimes_{\mathbb{Z}} mathbbZ) = \dim_{\mathbb{Q}}(H_1(G, \mathbb{Q})), \tag{1}$$

where $G^{ab} = G/[G, G]$ denotes the *abelianization* of G, and

$$d(G^{ab}) = \text{rank}_{\mathbb{Z}}(G^{ab}) = \text{rank}_{\mathbb{Z}}(H_1(G, \mathbb{Z})). \tag{2}$$

In particular, $\text{rank}_{\mathbb{Q}}(G) \leq d(G^{ab}) \leq d(G)$. The main purpose of this poster is to give an affirmative answer to the analogue of the Question above for the rational rank

Theorem. Let G be a non-abelian limit group, and let U be a normal subgroup of G of prime index p. Then $\text{rank}_{\mathbb{Q}}(U) > \text{rank}_{\mathbb{Q}}(G)$.

Representations of $\mathbb{Z} \wr \mathbb{Z}$

Tulio Marcio Gentil dos Santos (Universidade de Brasília - Brazil)

Abstract:
A group G is said to be self-similar if admits a faithful representation on a regular one-rooted m-tree $\mathcal{T}_m$ such that the representation is state-closed and is transitive on the tree's first level. In 2016, Dantas and Sidki [1] showed that $\mathbb{Z} \wr \mathbb{Z}$ cannot be self-similar. Although $\mathbb{Z} \wr \mathbb{Z}$ cannot be self-similar, recently we show that the group $\mathbb{Z} \wr \mathbb{Z}$ is state-closed of degree 3 and finite-state.

References

[1] A. C. Dantas and S. N. Sidki, *On self-similarity of wreath products of abelian groups.* arXiv:1610.08994, to appear in Groups, Geometry and Dynamics.

[2] A. M. Brunner and S. N. Sidki, *Abelian state-closed subgroups of automorphisms of m-ary trees.* Groups, Geometry, and Dynamics, **4** (2010) 455 - 471.

Braid groups

Mirele Pereira da Silva (Universidade Estatual de Feira de Santana - Brazil)

Abstract:
This poster aims to present a bit of my research on the Artin Braid Groups, still in progress. We will talk about their geometric definitions and why they actually form a group. We will also justify its usual finite presentation and prove some of its properties. This work is being guided by Kisnney Almeida and supported by an undergraduate scholarship PROBIC/UEFS.

Finite groups and their coprime automorphisms

Danilo Sanção da Silveira (Universidade Federal de Catalão - Brazil)

Abstract:
The purpose of this poster is to present new achievements on following general problem.

Problem: Let A be a group of automorphisms of a group G. What is the influence of the structure of centralizers $C_G(a)$, where $a \in A \setminus \{1\}$, has over the structure of G?

Let A be a suitable elementary abelian group acting coprimely on a finite or profinite group G. We show that if for all $a \in A \setminus \{1\}$ the elements in $\gamma_k(C_G(a))$ satisfy some natural Engel conditions, then $\gamma_k(G)$ satisfies similar conditions. Analogous results for word δ_k are also obtained.

Profinite topology in right-angled Artin groups

Saimon de Souza Rocha (Universidade Federal da Bahia - Brazil))

Abstract:
In this poster I will present some results on the profinite topology of right-angled artin groups, by Metaftsis and Raptis. This is a work in progress, part of my studies in masters program with Kisnney Almeida as advisor.

Automorphisms of the category of the finitely generated free groups of some subvariety of the variety of all groups

Ruan Barbosa Fernandes (Universidade Federal do Rio Grande do Norte - Brazil)

Abstract:
An important question in universal algebraic geometry is "When two universal algebras of the same variety have the same geometry?" Two concepts of universal algebraic geometry answer this question: the geometric equivalence and the automorphic equivalence. The relations between these two equivalents in an arbitrary variety of universal algebras Θ are studied by considering the category Θ^0 of all the finitely generated free algebras of Θ and their homomorphisms. When $\mathfrak{A}$ is the automorphism group of all the automorphisms of Θ^0 and $\mathfrak{Y}$ is the group of all the internal automorphisms of Θ^0, the quotient group $\mathfrak{A}/\mathfrak{Y}$ measures the possible difference between equivalences.

In [1] the variety $\Theta = Gr$ of all the groups was considered. It has been proven in this case that $\mathfrak{A}/\mathfrak{Y}$ is trivial. It is clear that when we consider $\Theta = Ab$ variety of all the commutative groups also $\mathfrak{A}/\mathfrak{Y}$ is trivial.

In [2] the $\mathfrak{N}_d$ varieties of all the nilpotent groups with the degree of nilpotence less than or equal to d ($d \geq 2$) were considered. In all these varieties also $\mathfrak{A}/\mathfrak{Y}$ is a trivial group. B. Plotkin asked whether there exists a $\Theta \subset Gr$ subvariety in Gr such that in this variety the group $\mathfrak{A}/\mathfrak{Y}$ is not trivial. A. Tsurkov suggested that in the subvariety of periodic groups, i.e., groups which fulfill identity $x^n = 1$, for some fixed $n > 2$) the group $\mathfrak{A}/\mathfrak{Y}$ may be not trivial.

We consider the subvariety $\Theta \subset Gr$ of all the groups with exponent 4, nilpotent of degree 4 or less and metabelian, i.e., the subvariety Θ which is defined in Gr by identities $x^4 = 1$, $((((x_1, x_2), x_3), x_4), x_5) = 1$, $((x_1, x_2), (x_3, x_4)) = 1$. We prove, that in this variety the group $\mathfrak{A}/\mathfrak{Y}$ is isomorphic to $\mathbb{Z}/2\mathbb{Z}$. Joint work with A. Tsurkov.

References

[1] B. Plotkin, G. Zhitomirski, Automorphisms of categories of free algebras of some varieties, *Journal of Algebra,* Volume 306, Issue 2, 2006, Pages 344-367.

[2] A. Tsurkov, Automorphisms of the category of the free nilpotent groups of the fixed class of nilpotency, *Int. J. Algebra Comput.,* Volume 17, Issue 5&6,2007, 1273-1281.