Space-time percolation and detection by mobile nodes

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Consider a Poisson point process of intensity $\lambda \in \mathbb{R}^d$. We denote the points as nodes and let each node move as an independent Brownian motion. Consider a target particle that is initially placed at the origin at time 0 and can move according to any continuous function. We say that the target is detected at time $t$ if there exists at least one node within distance 1 of the target at time $t$. We show that if $\lambda$ is sufficiently large, the target will eventually be detected even if its motion can depend on the past, present and future positions of the nodes. In the proof we use coupling and multi-scale analysis to model this event as a fractal percolation process and show that a good event percolates in space and time.