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Continuous time, Discrete Space mean field games

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I will present a recent joint work with Diogo A. Gomes (IST-Lisboa) and Joana Mohr (UFRGS) where we study a continuous time, discrete state space model for mean field games. Such kind of models can describe situations of competition between a large number of players, where all that we control is the statistical distribution of players and its strategies. After presenting our model, and introducing the concept of Nash equilibria for this games, I use tools from stochastic optimal control theory to derive forward and backwards equations describing the statistical distributions of players as well as the value function as functions of time. Considering that we are working in continuous time and discrete state space, such equations (Kolmogorov and Hamilton-Jacobi) are ODE´s connected to initial conditions for the distribution of players and terminal conditions for the value functions, which will lead to an initial-terminal value problem (ITVP), somewhat non standard in the ODE theory. We will prove the existence of solutions and discuss uniqueness for this ITVP, and, if time allows, show that, for short times, the mean field model can be approximated (in L^2 sense) by a N-player game, where the number N of players is large but finite.