A Fleming-Viot process driven by sub-critical branching: a selection principle

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Let X be a continuous time Markov chain in a countable state space S with an absorbing state that we call 0. In this context it is natural to study the distribution of the process conditioned on non-absorption, which we call the conditioned evolution. A quasi-stationary distribution (QSD) is a probability measure that is invariant for the conditioned evolution. These distributions are important since they represent the state of the process at large times for a typical path that has not been absorbed. The Fleming-Viot process associated to X (FV) is a particle system with state space S^N . At the beginning there are N (particles) independent copies of X, but when a particle is absorbed, it jumps instantaneously over one of the N-1 particles that are not absorbed at that time. After that time, each particle evolves independently of the others. The empirical measure of FV mimics the conditioned evolution of X. This fact makes FV a natural candidate to simulate the conditioned evolution for large times and also quasi-stationary distributions. We consider X to be a sub-critical branching process. In this case there is an infinite number of QSD, but there is a minimal one that has the smallest mean absorption time. We will prove that (i) the empirical measure of FV converges to the conditioned evolution in compact time intervals, (ii) FV is ergodic for every N (iii) the empirical measure of FV under its own invariant measure converges to the minimal QSD. That is, FV selects the minimal QSD. As a consequence, we also find the asymptotic behavior of a tagged particle in and out of equilibrium.

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