Introduction to Implicative Fuzzy Associative Memories

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Abstract

We introduce a new model of associative memory called *Implicative Fuzzy Associative Memory* (IFAM). This model is based on *implicative fuzzy learning*. IFAM model can be used to implement fuzzy systems that have a wide range of applications in control, prediction, etc. In the autoassociative case, the IFAM exhibits one pass convergence, unlimited storage capacity, and tolerance with respect to eroded patterns. These properties also characterize *Morphological Associative Memories* (MAM's). In fact, the MAM model can be viewed as a particular model of IFAM called *Lukasiewicz IFAM*. This model exhibited the best performance in a number of preliminary experiments.

1 Basic Concepts

Implicative Fuzzy Associative Memories (IFAM) are single layer feedforward fuzzy neural networks given by:

$$\mathbf{y} = W \circ \mathbf{x}, \tag{1}$$

where:

- $\mathbf{x} \in [0,1]^n$ is the input pattern,
- $W \in [0,1]^{m \times n}$ is the synaptic weight matrix,
- $\mathbf{y} \in [0,1]^m$ is the output,
- $\circ$ represents max-t composition:

$$y_i = \bigvee_{j=1}^{n} w_{ij} \,\mathrm{t}\, x_j \,.$$

1.1 Implicative Fuzzy Learning

Implicative fuzzy learning is used to store a set of associations:

$$\{\left(\mathbf{x}^\xi, \mathbf{y}^\xi\right) : \xi = 1, \ldots, p\} \,.$$

The synaptic weight change is given by a minimum of fuzzy implication:

$$w_{ij} = \bigwedge_{\xi=1}^{p} \left(x_j^\xi \Rightarrow y_i^\xi\right). \tag{2}$$

Interpretation: The synaptic weight change is the degree of truth of the sentence: "IF stimulus, THEN response".

In matrix form, we have

$$W = Y \circledast X^T, \tag{3}$$

where

- $X = [\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^p]$ is the matrix whose columns are the input patterns,
- $Y = [\mathbf{y}^1, \mathbf{y}^2, \ldots, \mathbf{y}^p]$ is the matrix whose columns are the output patterns,
- The matrix product $C = A \circledast B$ is computed as follows:

$$c_{ij} = \bigwedge_{l=1}^{k} \left(b_{lj} \Rightarrow a_{il}\right). \tag{4}$$

In particular, we speak of *R-implicative fuzzy learning* if an R-implication is used (in this case, $W = Y \circledast_R X^T$). The R-implication associatied with the t-nom t is given by:

$$(x \Rightarrow_R y) = \sup\{z \in [0,1] : x \,\mathrm{t}\, z \leq y\}. \tag{5}$$

2 Implicative Fuzzy Associative Memories

The associative memory problem consists of finding a mapping G such that $G(\mathbf{x}^\xi) = \mathbf{y}^\xi$. The mapping G should be endowed with a certain noise tolerance. In neuro-fuzzy associative memories, the mapping G is a fuzzy neural network. We obtain an *implicative fuzzy associative memory* (IFAM) when implicative fuzzy learning is used.

Given an input $\mathbf{x}$, the IFAM computes

$$\mathbf{y} = W \circ \mathbf{x}, \tag{6}$$

where

$$W = Y \circledast_R X^T. \tag{7}$$

2.1 IFAM x MAM

The *Lukasiewicz IFAM* $W = Y \circledast X^T$ uses the following t-norm and R-implication:

$$x \,\mathrm{t}\, y = 0 \vee (x + y - 1) \quad \text{and} \quad x \Rightarrow_R y = 1 \wedge (1 - x + y)\,. \tag{8}$$

The Lukasiewicz IFAM W and *Morphological Associative Memory* (MAM) W_{XY} are related in terms of the following equations:

$$W = (W_{XY} \wedge 0) + 1, \quad \text{and} \quad W \circ \mathbf{x} = [(W_{XY} \wedge 0) \boxslash \mathbf{x}] \vee 0\,. \tag{9}$$

2.2 Application

Fuzzy associative memories can be used to forecast the manpower requirement in a steel industry (Choudhury *et. al.*, 2002).

Method	Average Error
Lukasiewcz IFAM	2.29%
Kosko's FAM	2.67%
ARIMA1	9.79%
ARIMA2	5.48%

Table 1: Average errors in forecasting manpower.

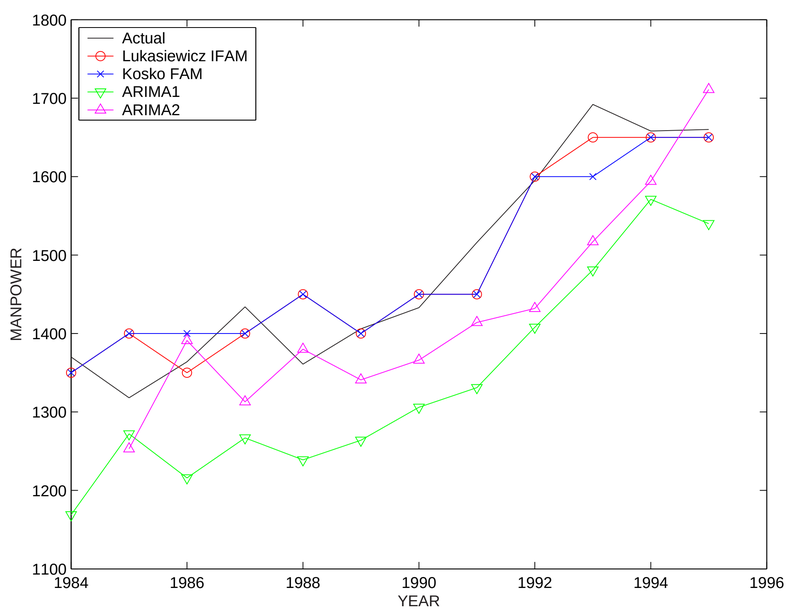

Figure 1: Comparision in forecasting manpower.

2.3 Dual IFAM

A dual formulation, called *Dual IFAM*, is described by:

$$\mathbf{y} = M \bullet \mathbf{x} \quad \text{and} \quad M = Y \bar{\circledast}_R Y^T, \tag{10}$$

where $\bullet$ represents min-s composition and $D = A \bar{\circledast}_R B$ is given by:

$$d_{ij} = \bigvee_{l=1}^{k} \left(b_{lj} \overline{\Rightarrow}_R a_{il}\right), \quad \text{and} \quad (x \overline{\Rightarrow}_R y) = \inf\{z \in [0,1] : x \,\mathrm{s}\, z \geq y\}.$$

Every statement on IFAM's induces a corresponding dual statement concerning Dual IFAM's in view of

$$A \bullet B = \overline{\bar{A} \circ \bar{B}} \quad \text{and} \quad A \bar{\circledast}_R B = \overline{\bar{A} \circledast_R \bar{B}}\,, \tag{11}$$

where $\bar{a}_{ij} = 1 - a_{ij}$ is the i,j-th compoment of $\bar{A}$.

3 Autoassociative Fuzzy Implicative Memories

The weight matrix of an *Autoassociative Fuzzy Implicative Memory* (AFIM) is given by:

$$W = X \circledast_R X^T. \tag{12}$$

Basic properties of AFIM's are:

- *One-pass convergence* (no feedback necessary).
- *Unlimited storage capacity* (in the absolute sense).
- *Tolerance with respect to eroded patterns*: The recalled pattern $\mathbf{y} = W \circ \mathbf{x}$ is the smallest fixed point of W such that $\mathbf{y} \geq \mathbf{x}$ (A pattern $\tilde{\mathbf{x}}$ is called an *eroded* version of a pattern $\mathbf{x}$ if and only if $\tilde{\mathbf{x}} \leq \mathbf{x}$).
- *Large number of fixed points*. Every expression in the original patterns $\mathbf{x}^\xi$

$$\mathbf{y} = \bigvee_{i=1}^{k} \bigwedge_{\xi \in L_i} \mathbf{x}^\xi, \tag{13}$$

is a fixed points of W.

3.1 Example

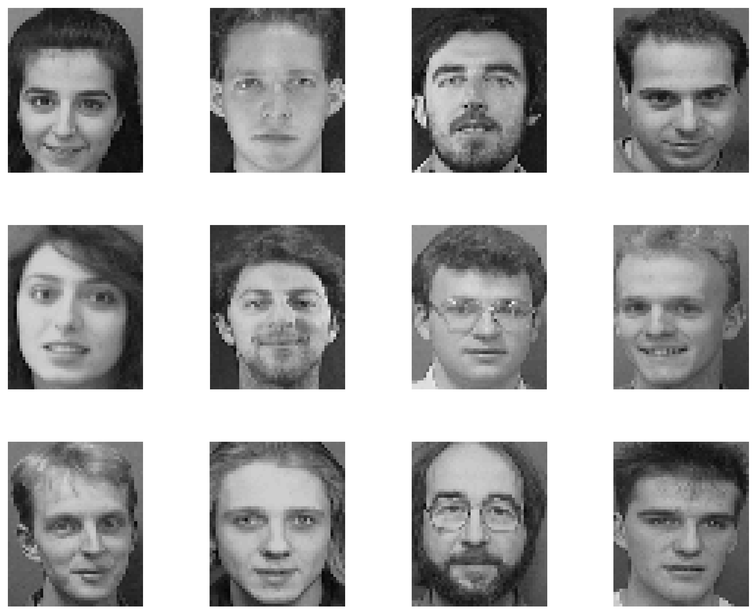

Figure 2: Stored Patterns.

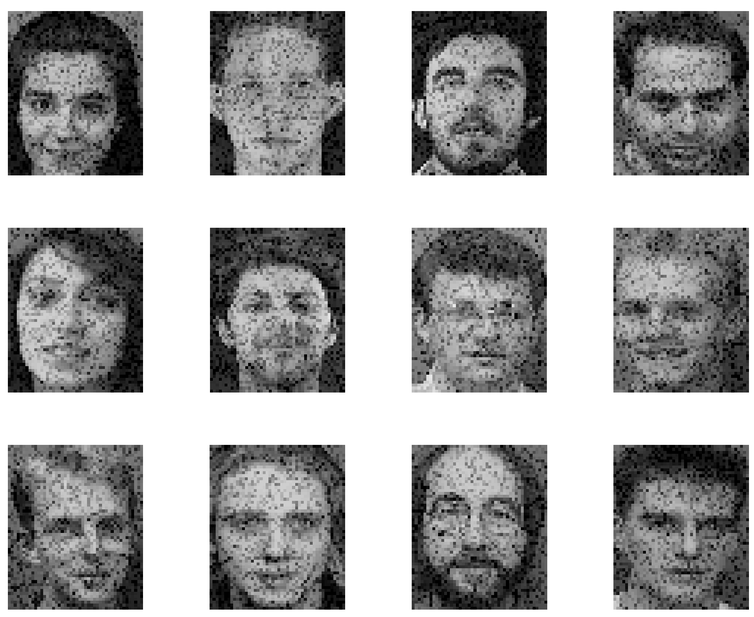

Figure 3: Patterns with erosive noise used as input.

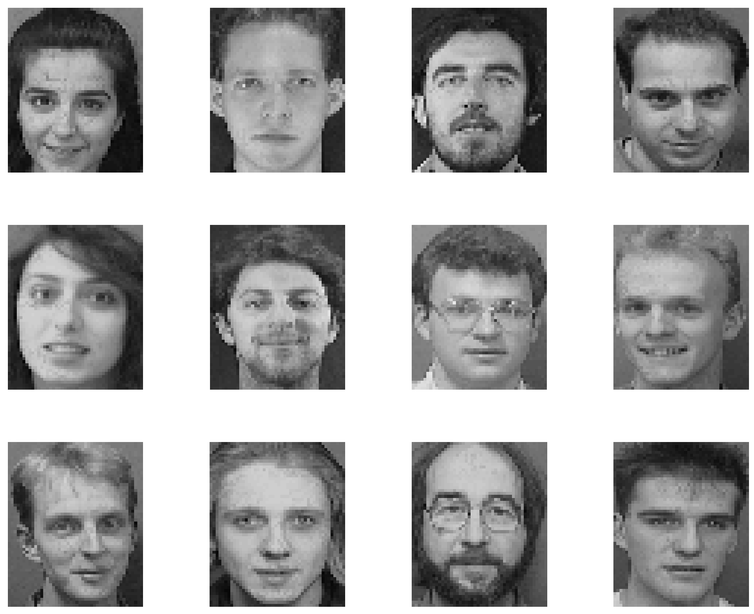

Figure 4: Patterns retrieved by Lukasiewicz AFIM when the input are the patterns of Figure 3.

Conclusion and Remarks

We introduced the *IFAM* and the *Dual IFAM* model.

We presented an application of IFAM model as a *fuzzy rule-based system* (Application 2.1).

We proved that AFIM's are characterized by *one pass convergence, unlimited storege capacity, tolerance with respect to eroded patterns, and a large number of fixed points*.

We showed that *Lukasiewicz IFAM* W and the *MAM* W_{XY} are *closely related*.

Suggestion for Further Research

1. *Properties of Heteroassociative* IFAM's.
2. *Mathematical foundations* of the IFAM model.
3. Selection of *the best IFAM model* for a given problem.
4. Does the IFAM represent a *universal approximator*?

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The patterns used in example 3.1 can be found in the eletronic address: `http://www.uk.research.att.com/facedatabase.html`.