Abstract
We introduce a new model of associative memory called Implicative Fuzzy Associative Memory (IFAM). This model is based on implicative fuzzy learning. IFAM model can be used to implement fuzzy systems that have a wide range of applications in control, prediction, etc. In the autoassociative case, the IFAM exhibits one pass convergence, unlimited storage capacity, and tolerance with respect to eroded patterns. These properties also characterize Morphological Associative Memories (MAMs). In fact, the MAM model can be viewed as a particular model of IFAM called Lukasiewicz IFAM. This model exhibited the best performance in a number of preliminary experiments.

1 Basic Concepts
Implicative Fuzzy Associative Memories (IFAM) are single layer feedforward fuzzy neural networks given by:

\[ y = W \circ x, \]

where:
- \( x \in [0, 1]^n \) is the input pattern,
- \( W \in [0, 1]^{m \times n} \) is the synaptic weight matrix,
- \( y \in [0, 1]^m \) is the output,
- \( \circ \) represents max-t composition:

\[ y_i = \bigwedge_{j=1}^n w_{ij} \circ x_j. \]

1.1 Implicative Fuzzy Learning
Implicative fuzzy learning is used to store a set of associations:

\[ \{(x^i, y^i) \colon i = 1, \ldots, p\}. \]

The synaptic weight change is given by a minimum of fuzzy implication:

\[ w_{ij} = \bigvee_{z=1}^n \{x^i \circ y^i \}, \]

Interpretation: The synaptic weight change is the degree of truth of the sentence: "IF stimulus, THEN response".

In matrix form, we have

\[ W = Y \circ X^T, \]

where:
- \( X = (x^1, x^2, \ldots, x^p) \) is the matrix whose columns are the input patterns,
- \( Y = (y^1, y^2, \ldots, y^p) \) is the matrix whose columns are the output patterns,
- The matrix product \( C = A \circ B \) is computed as follows:

\[ c_{ij} = \bigvee_{k=1}^n (a_{ik} \circ b_{kj}). \]

In particular, we speak of R-implicative fuzzy learning if an R-implication is used in (this case, \( W = Y \circ R X^T \)). The R-implication associated with the t-norm \( \ast \) is given by:

\[ (x \ast_R y) = \sup \{ z \in [0, 1] \colon x \circ z \leq y \}. \]

2 Implicative Fuzzy Associative Memories
The associative memory problem consists of finding a mapping \( C \) such that \( C(x^i) = y^i \). The mapping \( C \) should be endowed with a certain noise tolerance. In neuro-fuzzy associative memories, the mapping \( C \) is a fuzzy neural network. We obtain an implicative fuzzy associative memory (IFAM) when implicative fuzzy learning is used.

Given an input \( x \), the IFAM computes

\[ y = W \circ x, \]

where

\[ W = Y \circ R X^T. \]

2.1 IFAM x MAM
The Lukasiewicz IFAM \( W = Y \circ X^T \) uses the following t-norm and R-implication:

\[ x \ast_R y = \sup \{ z \in [0, 1] \colon x \circ z \leq y \} \quad \text{and} \quad x \ast y = x \land (1 - x + y). \]

The Lukasiewicz IFAM \( W \) and Morphological Associative Memory (MAM) \( W_{XY} \) are related in terms of the following equations:

\[ W = (W_{XY} \land 0) + 1, \quad \text{and} \quad W \ast x = (W_{XY} \circ 0) \bigvee \land 0. \]

2.2 Application
Fuzzy associative memories can be used to forecast the manpower requirement in a steel industry (Choudhury et al., 2002).

<table>
<thead>
<tr>
<th>Method</th>
<th>Average Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lukasiewicz IFAM</td>
<td>2.5%</td>
</tr>
<tr>
<td>Kosko’s FAM</td>
<td>2.67%</td>
</tr>
<tr>
<td>ARIMA1</td>
<td>9.79%</td>
</tr>
<tr>
<td>ARIMA2</td>
<td>5.46%</td>
</tr>
</tbody>
</table>

2.3 Dual IFAM
A dual formulation, called Dual IFAM, is described by:

\[ y = M \circ x \quad \text{and} \quad M = Y \circ R X^T, \]

where \( \circ \) represents min-s composition and \( D = A \circ B \) is given by:

\[ d_{ij} = \bigwedge_{k=1}^n (b_{kj} \circ a_{ik}), \quad \text{and} \quad (a \equiv_R b) = \inf \left\{ \{b \in [0, 1] : x \circ_z y \geq b\} \right\}. \]

Every statement on IFAM’s induces a corresponding dual statement concerning Dual IFAM’s in view of

\[ A \equiv B = A \circ R B \quad \text{and} \quad A \circ B = A \equiv_R B, \]

where \( a_{ij} = 1 - w_{ij} \) is the \( i,j \)-th component of \( A \).

3 Autoassociative Fuzzy Implicative Memories
The weight matrix of an Autoassociative Fuzzy Implicative Memory (AFIM) is given by:

\[ W = X \circ_R X^T. \]

Basic properties of AFIM’s are:
- One-pass convergence (no feedback necessary).
- Unlimited storage capacity (in the absolute sense).
- Tolerance with respect to eroded patterns: The recalled pattern \( y = W \circ x \) is the smallest fixed point of \( W \) such that \( y \geq x \). (A pattern \( x \) is called an eroded version of a pattern \( \tilde{x} \) if and only if \( x \leq \tilde{x} \).
- Large number of fixed points. Every expression in the original patterns \( x^i \):

\[ y = \bigvee_{i \in \mathbb{N}} x^i \]

is a fixed points of \( W \).

Conclusion and Remarks
We introduced the IFAM and the Dual IFAM model.

We presented an application of IFAM model as a fuzzy rule-based system (Application 2.1).

We proved that AFIM’s are characterized by one pass convergence, unlimited storage capacity, tolerance with respect to eroded patterns, and a large number of fixed points.

We showed that Lukasiewicz IFAM \( W \) and the MAM \( W_{XY} \) are closely related.

Suggestion for Further Research
1. Properties of Heteroassociative IFAM’s.
2. Mathematical foundations of the IFAM model.
4. Does the IFAM represent a universal approximator?