Associative Memories Based on Fuzzy Mathematical Morphology and an Application in Prediction

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Introduction

An *associative memory* (AM) is a model of the biological phenomena that allow for the storage of pattern associations. Furthermore, an AM allows for the retrieval of the desired output upon presentation of a possibly noisy or incomplete version of an input pattern.

Fuzzy Associative Memories and their Applications

A *fuzzy associative memory* (FAM) belongs to the class of *fuzzy neural networks* (FNNs). A FNN is an *artificial neural network* (ANN) whose input patterns, output patterns, and/or connection weights are fuzzy-valued.

Fuzzy associative memories are generally used to implement fuzzy rule-based systems. Applications of FAMs include backing up a truck and trailer, target tracking, human-machine interfaces, robot control, and voice cell control in ATM networks

Fuzzy Morphological Associative Memories

We recently observed that many well-known FAM models perform elementary operations of *fuzzy mathematical morphology* at every node. Therefore, many FAM models can be viewed as *fuzzy morphological associative memories* (FMAMs) [3].

FMAMs involve concepts from the areas of mathematical morphology, fuzzy set theory, and artificial neural networks. Consequently, they can be explored from different points of view. For example, we developed a general *learning strategy* for FMAMs that is based on the notion of *adjunction*. We intend to investigate other aspects of FMAMs in the near future.

Basic Concepts on FMAMs

FMAMs are equipped with *fuzzy morphological neurons*, i.e., fuzzy neurons that perform either a *dilation*, an *erosion*, a *anti-dilation*, or an *anti-erosion* [2, 3]. Recall that every mapping between complete lattices can be expressed in terms of supremums and infimums of these four operations [1].

Some Basic Concepts on Fuzzy Sets Theory

Fuzzy morphological neurons are usually defined in terms of fuzzy conjunctions and fuzzy disjunctions.

A *fuzzy conjunction* is an increasing mapping $C : [0,1]\times[0,1] \rightarrow [0,1]$ that satisfies $C(0,0) = C(0,1) = C(1,0) = 0$ and $C(1,1) = 1$. Examples of fuzzy conjunctions include: $C_M(x,y) = x \wedge y$ (minimum), $C_P(x,y) = xy$ (product), and $C_L(x,y) = 0 \vee (x+y-1)$ (Lukasiewicz).

A *fuzzy disjunction* is an increasing mapping $D : [0,1]\times[0,1] \rightarrow [0,1]$ that satisfies $D(0,0) = 0$ and $D(0,1) = D(1,0) = D(1,1) = 1$. Examples of fuzzy disjunctions include: $D_M(x,y) = x \vee y$ (maximum), $D_P(x,y) = x+y-x\cdot y$ (probabilistic sum), and $D_L(x,y) = 1 \wedge (x+y)$ (Lukasiewicz).

A fuzzy conjunction and a fuzzy disjunction can be linked by means of a duality relationship such as *negation* or *adjunction*.

Max-C and Min-D Fuzzy Matrix Products

The fuzzy operations C and D can be combined with the maximum or the minimum operation to yield the following matrix products.

We define the *max-C product* and the *min-D products* of $A \in [0,1]^{m\times k}$ and $B \in [0,1]^{k\times n}$, denoted by $G = A \circ B$ and $H = A \bullet B$, as follows:

$$g_{ij} = \bigvee_{\xi=1}^{k} C(a_{i\xi}, b_{\xi j}) \quad \text{and} \quad h_{ij} = \bigwedge_{\xi=1}^{k} D(a_{i\xi}, b_{\xi j}) \,. \tag{1}$$

Max-C Morphological Neurons

The max-C morphological neurons represent the most important types of fuzzy neurons that occur in FMAM models.

If $\mathbf{x} = [x_1,\ldots,x_n]^T$ represents the fuzzy inputs and $\mathbf{w} = [w_1,\ldots,w_n]^T$ corresponds the fuzzy synaptic weights of a *max-C neuron* then we compute the output $y \in [0,1]$ as follows:

$$y = \bigvee_{j=1}^{n} C(w_j, x_j) = \mathbf{w}^T \circ \mathbf{x} \,. \tag{2}$$

We speak of a *max-C morphological neuron* if and only if $C(x,\cdot)$ is a dilation for every $x \in [0,1]$. In this case, Equation 2 corresponds to a fuzzy dilation since it commutes with the supremum [3, 4].

Max-C FMAMs

Several well-known FAM models can be viewed as single layer feedforward ANNs with max-C morphological neurons. These models are given by the equation

$$\mathbf{y} = \mathcal{W}(\mathbf{x}) = W \circ \mathbf{x} \,, \tag{3}$$

where $W \in [0,1]^{m\times n}$ represents the synaptic weight matrix and $\mathbf{x} \in [0,1]^n$ and $\mathbf{y} \in [0,1]^m$ are the fuzzy input and fuzzy output patterns. The operator $\mathcal{W}$ represents a dilation from $[0,1]^n$ into $[0,1]^m$.

Examples of max-C FMAMs include the FAMs of Kosko, Junbo et al., Liu, Chung and Lee, and the IFAM models [3].

The Dual Model: Min-D FMAM

A max-C FMAM corresponds to a dilation $\mathcal{W}$. Thus, the relationships of duality with respect to a negation or adjunction can be used to formulate new FMAM models. These dual models perform an erosion and can be viewed as single layer feedforward ANNs with min-D morphological neurons.

A min-D neuron is defined by the equation

$$y = \bigwedge_{j=1}^{n} D\left(m_j, x_j\right) = \mathbf{m}^T \bullet \mathbf{x} \,, \tag{4}$$

where $\mathbf{x} = [x_1,\ldots,x_n]^T$ represents the input, $\mathbf{m} = [m_1,\ldots,m_n]^T$ corresponds to the synaptic weights, and y is the output.

Examples of min-D FMAMs include FLBAMs and dual IFAM [3].

Fuzzy Learning by Adjunction

Fuzzy learning by adjunction yields a general class of recording strategies for max-C FMAMs [4].

Suppose that we want to store a set of input-output pairs $\{(\mathbf{x}^\xi, \mathbf{y}^\xi) : \xi = 1,\ldots,k\}$ in a max-C FMAM given by Equation 3. For simplicity, let $X \in [0,1]^{n\times k}$ and $Y \in [0,1]^{m\times k}$ denote the matrices whose columns are the vectors $\mathbf{x}^\xi$ and $\mathbf{y}^\xi$, respectively.

Define the operator $\mathcal{D}_X : [0,1]^{m\times n} \rightarrow [0,1]^{m\times k}$ as follows:

$$\mathcal{D}_X(W) = W \circ X \,. \tag{5}$$

If $\mathcal{D}_X$ is a fuzzy dilation, then there exists an unique fuzzy erosion $\mathcal{E}_X : [0,1]^{m\times k} \rightarrow [0,1]^{m\times n}$ that forms an adjunction with $\mathcal{D}_X$. Thus, the following equation can be used to define a synaptic weight matrix for a max-C FMAM:

$$W = \mathcal{E}_X(Y) \,. \tag{6}$$

This synaptic weight matrix satisfies several interesting properties. For example, W given by Equation 6 represents the best approximation from below of Y in terms of the max-C product [4].

Finally, an application of the duality relationship of negation to the synaptic weight matrix of a max-C FMAM leads to a recording scheme for min-D FMAMs.

An Application in Prediction

Consider the problem of forecasting the *monthly streamflow of a large hydroelectric plant*, called Furnas. The predictions are used for simulation, optimization, and decision-making purposes of the energy system [2].

A time series prediction problem can be formulated as follows: Given samples s_μ for $\mu = 1,\ldots,q-1$, obtain an estimate $\hat{s}_q$ for the actual s_q based on a subset of the past values $s_1,\ldots,s_{q-1}$.

Description of our Approach

Our approach is based on the following items:

- We used the Lukasiewicz FMAM since it outperformed other FAM models in a previous work;
- We adopted 12 different models, one for each month of the year, due to the seasonality of the streamflow;
- A fixed number of three antecedents. For example, the values of January, February, and March were taken into account to predict the streamflow of April.

An FMAM can be used as follows in order to predict a time series:

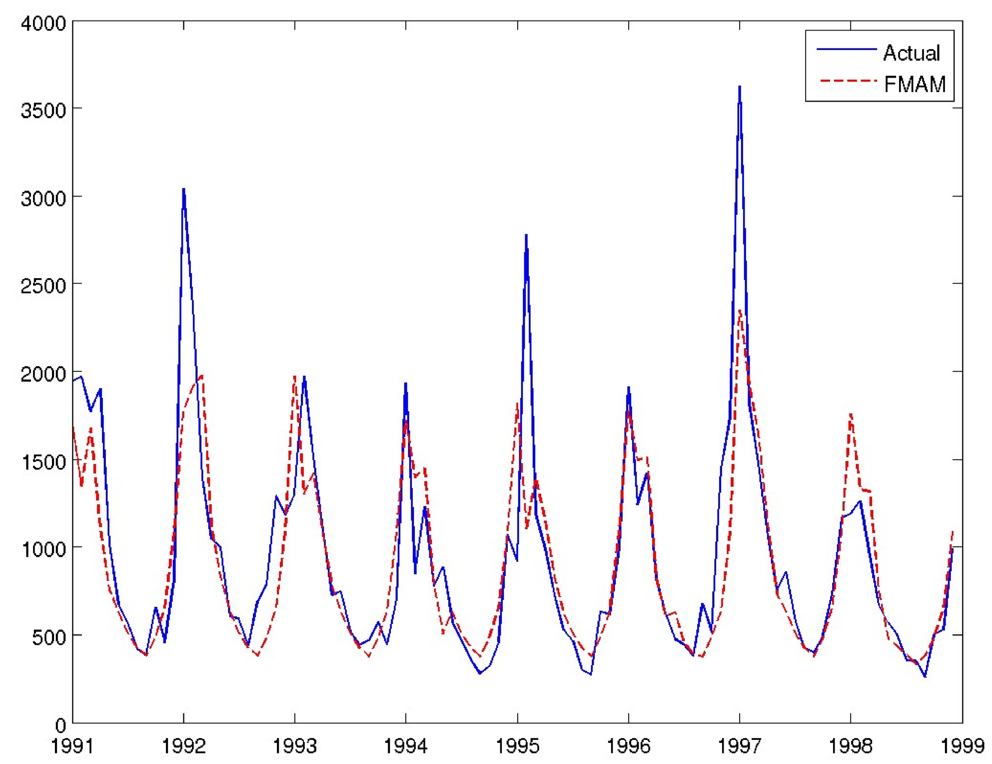
1. Define fuzzy sets $\mathbf{x}^\xi$ and $\mathbf{y}^\xi$ that comprise some relevant information concerning the past values of the time series.
2. Store the associations $(\mathbf{x}^\xi, \mathbf{y}^\xi)$ in the memory;
3. Given an input $\mathbf{x}^q$ that takes into account some past values, compute the output pattern $\mathbf{y}^q$;
4. A defuzzification of $\mathbf{y}^q$ yields the estimation $\hat{s}_q$.

These fours steps were performed as follows:

1. We used the *subtractive clustering method* to determine fuzzy sets $\mathbf{x}^\xi$ and $\mathbf{y}^\xi$ with Gaussian-type membership functions from streamflow data from 1931 to 1990;
2. The *fuzzy learning by adjunction* was employed to store the associations in the FMAM (cf. Equation 6);
3. For computational reasons, we used a discrete Dirac-δ function to model the input pattern $\mathbf{x}^q$;
4. A defuzzification of $\mathbf{y}^q$ using the center of mass yielded $\hat{s}_q$.

Experimental Results

The following figure shows the predictions that we obtained for the time period from 1991 to 1998. The blue line corresponds to the actual values and the red dashed line corresponds to our prediction based on the Lukasiewicz FMAM with fuzzy learning by adjunction.



The following table compares the mean square (MSE), mean absolute (MAE), and mean relative percentage errors (MRPE) that were generated by the FMAM model and several other models.

Methods	MSE ($\times 10^5$)	MAE (m^3/s)	MRPE (%)
FMAM	1.38	221	21
PARMA	1.85	280	28
MLP	1.82	271	30
NFN	1.73	234	20
FPM-PRP	1.20	200	18

We would like to point out that the MLP (multi-layer perceptron), NFN (neuro-fuzzy network), and FPM-PRP (fuzzy prediction model based on a pattern recognition procedure) models were initialized by optimizing the number of the parameters for each monthly prediction. For example, the MLP considers 4 antecedents to predict the streamflow of January and 3 antecedents to predict the streamflow for February. Moreover, the FPM-PRP model also takes into account slope information which requires some additional “fine tuning”. We experimentally determined a variable number of parameters (including slopes) for the FMAM model such that MSE = 0.88×10^5, MAE = 157, and MPE = 15.

References

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