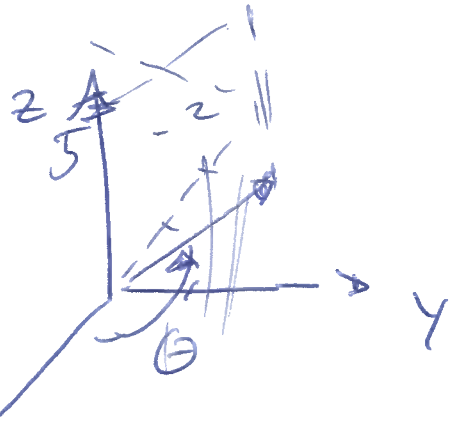
1. Uma parabola é um conjunto de pontos da forma
$\{P \in \mathbb{R}^2 \mid dist(P, F) = dist(P, R)\}$
para algum $F \in \mathbb{R}^2$ e alguma reta $R \subseteq \mathbb{R}^2$

2. Seja $P = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$



$r = \sqrt{(-2)^2 + 1^2} = \sqrt{5}$

$\theta = \measuredangle\left(E_1, \begin{pmatrix} -2 \\ 1 \end{pmatrix}\right) \in [0, 2\pi)$

Note que $y = 1 > 0$, portanto $\theta \in (0, \pi)$

$$\cos\theta = \frac{\langle E_1, \begin{pmatrix} -2 \\ 1 \end{pmatrix} \rangle}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

Então $\theta = \arccos\left(-\frac{2\sqrt{5}}{5}\right)$

coordenadas cilíndricas de P:

$$(r, \theta, z) = \left(\sqrt{5}, \arccos\left(-\frac{2\sqrt{5}}{5}\right), 5\right)$$

3.(a) Seja $\ell = S \cap P_{z=0}$

$= \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid 2(x^2+y^2+2xy) - y + x = 1 \right\}$

De feto, $\ell = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \mid \ldots\ldots \right\}$

Temos

$$(x\ y) \underbrace{\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix} + (1\ -1) \begin{pmatrix} x \\ y \end{pmatrix} = 1$$

Autovalores de A

$$0 = \det \begin{pmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{pmatrix} = (\lambda-2)^2 - 4$$
$$= \lambda^2 - 4\lambda + \cancel{4} - \cancel{4}$$

$\lambda_1 = 0,\ \lambda_2 = 4$ $\qquad = \lambda(\lambda - 4)$

$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ e $\lambda_1 \cdot \lambda_2 = 0$

Então se não fo conica degenerada

se trafer de uma parabola

(b) Autovetores de A

(i) $\lambda_1 = 0$: $\quad (A - \lambda I) V = 0$

$$\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad x = -y \Rightarrow y = -x$$

$$S = \left\{ \begin{pmatrix} x \\ -x \end{pmatrix} \mid x \in \mathbb{R} \right\}$$
$$= \left\{ \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix} \mid \alpha \in \mathbb{R} \right\}$$

$\underline{\lambda_2 = 4}$:

$$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \quad x = y$$

$$S = \left\{ \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix} \mid \alpha \in \mathbb{R} \right.$$

Seja $V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\bar{V}_1 = \dfrac{V_1}{\|V_1\|} = \dfrac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$V_2 = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(d) Base ortonormal de autovetores

$$\bar{B} = \{ \bar{V}_1, \bar{V}_2 \}$$

Seja $R = \begin{pmatrix} \bar{V}_1 & \bar{V}_2 \end{pmatrix} = \dfrac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

Temos $\det(R) = 1 \Rightarrow R$ é matriz de rotação

$$R = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \text{ com } \theta = -45^\circ$$

(e) Equação no sistema rodado $(\bar{x}, \bar{y})$:

$$(\bar{x}\ \bar{y})_{\bar{B}} \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_{\bar{B}} + (1\ -1) \underbrace{I^{\bar{B}}_{B} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_{\bar{B}}}_{\begin{pmatrix} x \\ y \end{pmatrix}} = 1$$

$$\Leftrightarrow (\bar{x}\ \bar{y})_{\bar{B}} \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_{\bar{B}} + (1\ -1) \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_{\bar{B}} = 1$$

$$\Leftrightarrow 4\bar{y}^2 + \frac{\sqrt{2}}{2}(2\ 0)\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_{\bar{B}} = 1$$

$$\Leftrightarrow 4\bar{y}^2 + \sqrt{2}\,\bar{x} = 1$$

$$\Leftrightarrow 4\bar{y}^2 = 1 - \sqrt{2}\,\bar{x}$$

$$\Leftrightarrow \bar{y}^2 = -\frac{1}{4}(\sqrt{2}\,\bar{x} - 1)$$

$$\Leftrightarrow \underbrace{\bar{y}^2}_{\hat{y}} = \underbrace{-\frac{\sqrt{2}}{4}}_{4p} \underbrace{\left(\bar{x} - \frac{\sqrt{2}}{2}\right)}_{\hat{x}}$$

(c)

$$p = -\frac{\sqrt{2}}{16}$$

Foco em $(\hat{x}, \hat{y})$: $\begin{pmatrix} -\frac{\sqrt{2}}{16} \\ 0 \end{pmatrix}_{\hat{x}, \hat{y}}$

" $(\bar{x}, \bar{y})$ $\left(\bar{x} = \hat{x} + \frac{\sqrt{2}}{2}\right)$: $\begin{pmatrix} \frac{7\sqrt{2}}{16} \\ 0 \end{pmatrix}$

Mudança de base através de $I_{\mathcal{B}}^{\bar{\mathcal{B}}} = R$

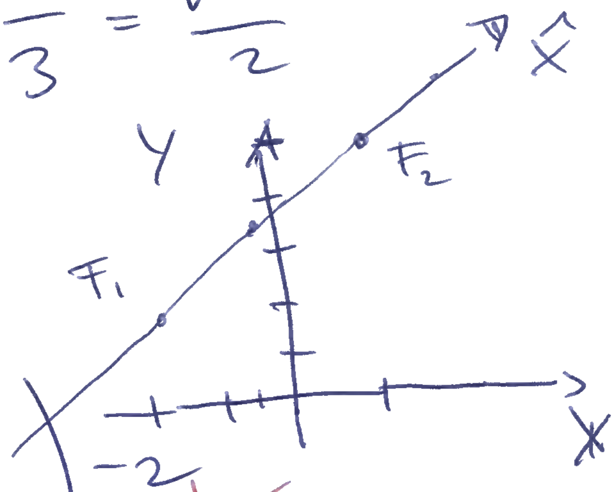
Foco em (x,y): $\frac{\sqrt{2}}{2}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \frac{7\sqrt{2}}{16} \\ 0 \end{pmatrix}$

$= \frac{7}{16}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{7}{16}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

4.(a) $2c = \text{dist}(F_1, F_2) = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$\Rightarrow c = 3\frac{\sqrt{2}}{2}$

$3 = e = \frac{c}{a} \Rightarrow a = \frac{c}{3} = \frac{\sqrt{2}}{2}$



Seja C o centro

As coordenadas de C devem ser $\begin{pmatrix} \frac{1+(-2)}{2} \\ \frac{2+5}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{7}{2} \end{pmatrix}$

1/4

Note que $\overrightarrow{F_1F_2} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Seja D o vetor $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \in \mathbb{R}^2$

Para os vertices $A_{1,2}$ temos $A_{1,2} = C + \alpha_{1,2}$

para alguns

Temos $a = \text{dist}(C, A_2)$

$$= \|\alpha_2 D\| =$$

$$\Rightarrow \quad \alpha_2 \sqrt{2} = \frac{\sqrt{2}}{2} \Rightarrow \alpha_2 = \frac{1}{2}$$

Então $A_2 = C + \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\4\end{pmatrix}$ $\frac{1}{4}$

Além disso $\alpha_1 = -\alpha_2 = -\frac{1}{2}$

$$\Rightarrow A_1 = C - \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix}$$

Observe tb que

$$c^2 = a^2 + b^2$$

$$\Leftrightarrow 3^2 \cdot \frac{1}{2} = \frac{1}{2} + b^2$$ $\frac{1}{4}$

$$\Rightarrow \quad b^2 = \frac{1}{2}(9-1) = 4$$

$$\Rightarrow \quad b = 2$$

Eq's das assintotas no sistema $(\hat{x}, \hat{y})$:

$$\hat{y} = \pm \frac{b}{a} \hat{x} = \pm \frac{4}{\sqrt{2}} \hat{x}$$ $\frac{1}{4}$

Por ex. para $\hat{x} = \sqrt{2}$ temos $\hat{y} = \pm 4$

então para $x = -\frac{1}{2} + \sqrt{2}$, $y = 3{,}5 \pm 4$

Além disso, as assintotas passam pa $C = \begin{pmatrix} -\frac{1}{2} \\ \frac{7}{2} \end{pmatrix}$

Dado 2 pts podemos calcular a eq da reta que passa por estes pts.

Alternative: Note que $(\pm \frac{b}{a}, -1) \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = 0$

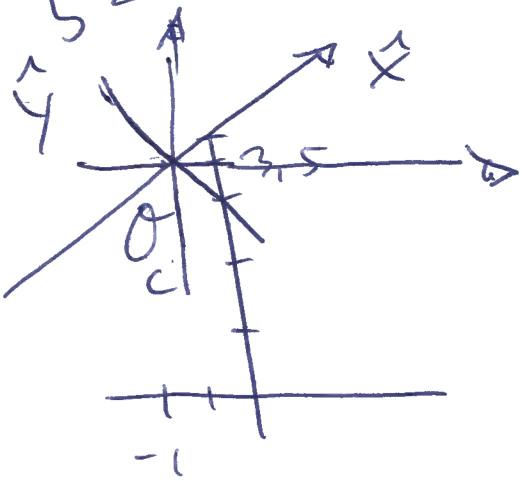
$\iff (\pm \frac{b}{a}, -1)\, I^{B}_{\hat{B}}\, \underbrace{I^{\hat{B}}_{B} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}}$ =

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x+ \\ y- \end{pmatrix}$

(b) Em $(\hat{x}, \hat{y})$ temos

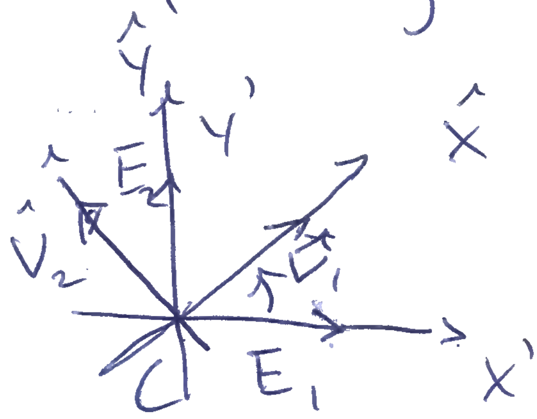
$$\frac{\hat{x}^2}{a^2} - \frac{\hat{y}^2}{b^2} = 1 \iff 2\hat{x}^2 - \frac{\hat{y}^2}{4} = 1$$ veja 4.e)

4.(c)



Seja $\hat{B} = \left\{ \frac{\sqrt{2}}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{\sqrt{2}}{2} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

e seja $B = \{E_1, E_2\}$



Note que $\theta = 45° = \frac{\pi}{4}$

e $R = (\bar{V}_1 \bar{V}_2) = \frac{\sqrt{2}}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

Para ir de $(\hat{x}, \hat{y})$ para (x', y')

aplicamos $I_{\mathcal{B}}^{\hat{\mathcal{B}}} = R$, i.e. $I_{\mathcal{B}}^{\hat{\mathcal{B}}} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}_{\hat{\mathcal{B}}} = \begin{pmatrix} x' \\ y' \end{pmatrix}$

Em $\hat{x}, \hat{y}$:

$$(\hat{x}\ \hat{y})_{\mathcal{B}} \underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -\frac{1}{4} \end{pmatrix}}_{D} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}_{\hat{\mathcal{B}}} - 1 = 0$$

Seja A t.q. $AR = RD \Leftrightarrow$ ~~Ae~~ $D = R^T A R$

$$= \left(I_{\mathcal{B}}^{\hat{\mathcal{B}}}\right)^T A\, I_{\mathcal{B}}^{\hat{\mathcal{B}}}$$

1/2

$$(\hat{x}\ \hat{y})_{\mathcal{B}} \begin{pmatrix} 2 & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}_{\hat{\mathcal{B}}} - 1 = 0$$

$$\Leftrightarrow \underbrace{\left(\cancel{I_{\mathcal{B}}^{\hat{\mathcal{B}}}}\right)^T (\hat{x}\ \hat{y})_{\hat{\mathcal{B}}}}_{\left(I_{\mathcal{B}}^{\hat{\mathcal{B}}}\right)^T} A \left(I_{\mathcal{B}}^{\hat{\mathcal{B}}} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}_{\hat{\mathcal{B}}}\right) = 1$$

$$\Leftrightarrow (x'\ y') A \begin{pmatrix} x' \\ y' \end{pmatrix} = 1$$

$$\Leftrightarrow (x+0{,}5\ \ y-3{,}5) A \begin{pmatrix} x+0{,}5 \\ y-3{,}5 \end{pmatrix} = 1$$

onde $A = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{8} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ \frac{1}{8} & -\frac{1}{8}\end{pmatrix} = \frac{1}{8}\begin{pmatrix}7 & 9\\ 9 & 7\end{pmatrix}$$

Eq. no sistema x, y:

$$(x+0{,}5 \quad y-3{,}5)\begin{pmatrix}7 & 9\\ 9 & 7\end{pmatrix}\frac{1}{8}\begin{pmatrix}x+0{,}5\\ y-3{,}5\end{pmatrix}$$

1/2

Continuacao 4, a):

Eq's das assintotas em (x,y)

Temos $\hat{y} = \pm\frac{4}{\sqrt{2}}\hat{x}$ no sist. rodado e transladado

$$\Leftrightarrow \left(\mp\frac{4}{\sqrt{2}}, 1\right)\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix} = 0$$

$$\Leftrightarrow \left(\mp\frac{4}{\sqrt{2}}, 1\right)\underbrace{I_{\hat{B}}^{B}}_{(I_B^{\hat{B}})^T}\underbrace{I_B^{\hat{B}}\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix}}_{\begin{pmatrix}x'\\ y'\end{pmatrix} = \begin{pmatrix}x+0{,}5\\ y-3{,}5\end{pmatrix}} = 0$$

$$\Leftrightarrow \left(\mp\frac{4}{\sqrt{2}}, 1\right)\frac{\sqrt{2}}{2}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}x+0{,}5\\ y-3{,}5\end{pmatrix} = 0$$

$$\Leftrightarrow (\mp 4, \sqrt{2})\begin{pmatrix}x+y-3\\ y-x-4\end{pmatrix} = 0$$

Se quiser pode escrever estas eq's na forma $y = \alpha x + \beta$

Questão adicional: Retas Diretrizes de $\mathcal{H}$

No sistema $(\hat{x},\hat{y})$, os focos estão em

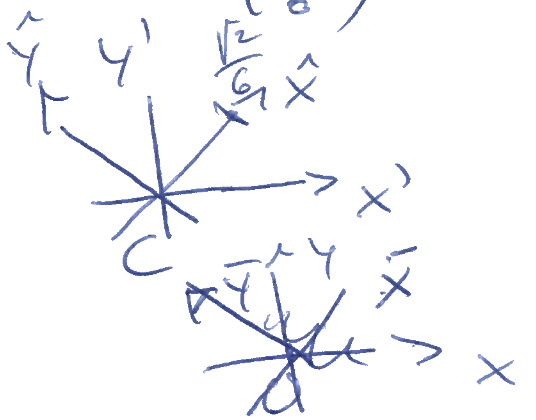
$\begin{pmatrix}\pm c\\0\end{pmatrix}$, onde $c = \frac{3\sqrt{2}}{2}$

Portanto, as retas diretrizes em $(\hat{x},\hat{y})$ tem as eq's $x = \pm\frac{c}{e^2} = \pm\frac{3\sqrt{2}}{2\cdot 9} = \pm\frac{\sqrt{2}}{6}$

Sejam D_1 e D_2 as retas diretrizes.

D_1 passa por $\begin{pmatrix}\frac{\sqrt{2}}{6}\\0\end{pmatrix}$ e D_2 passa por $\begin{pmatrix}-\frac{\sqrt{2}}{6}\\0\end{pmatrix}$ em $(\hat{x},\hat{y})$

Aplicando mudança de base e translação obtemos



$$I^{\hat{B}}_{B}\begin{pmatrix}\pm\frac{\sqrt{2}}{6}\\0\end{pmatrix}$$

$C = \begin{pmatrix}0\\0\end{pmatrix}_{\hat{x},\hat{y}}$ corresponde a $\begin{pmatrix}-0{,}5\\3{,}5\end{pmatrix}$ em $(\bar{x},\bar{y})$

$\begin{pmatrix}\pm\frac{\sqrt{2}}{6}\\0\end{pmatrix}$ corresponde