

$$1.(b) \quad I(x, y) = N(x \dot{+} N(y))$$

$$= 1 - [x \dot{+} (1 - y)]$$

$$= 1 - [0 \vee (x + 1 - y - 1)]$$

$$= 1 - [0 \vee (x - y)]$$

$$= 1 + [0 \wedge (-x + y)]$$

$$= 1 \wedge (1 - x + y)$$

$$= 1 \wedge [N(x) + y] = N(x) \text{ s } y$$

Pode também aplicar De Morgan para \vee e \wedge .

$$(c) \quad x \Rightarrow_R y = \bigvee \{z \in [0, 1] : x \dot{+} z \leq y\}$$

$$= \bigvee \{z \in [0, 1] : 0 \vee (x + z - 1) \leq y\}$$

$$= \bigvee \{z \in [0, 1] : 0 \leq y \text{ e } x + z - 1 \leq y\}$$

$$= \bigvee \{z \in [0, 1] : x + z - 1 \leq y\}$$

$$= \begin{cases} 1 & \text{se } x \leq y \\ 1 - x + y & \text{se } x > y (\Leftrightarrow y - x < 0) \end{cases}$$

$$= \begin{cases} 1 & \text{se } 1 - x + y \geq 1 \\ 1 - x + y & \text{se } 1 - x + y < 1 \end{cases}$$

$$= 1 \wedge (1 - x + y) = N(x) \text{ s } y = I(x, y)$$

onde $N(x) = 1 - x$ e $x \text{ s } y = 1 \wedge (x + y) \quad \forall x, y \in [0, 1]$