4.(a) Seja $\hat{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 3 & 2 \\ -3 & 4 & -5 \end{pmatrix}$, $\hat{b} = \begin{pmatrix} 5 \\ 6 \\ -3 \end{pmatrix}$

$Ax = b \iff \hat{A}x = \hat{b}$

$\hat{C} = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{3} & 0 & -\frac{2}{3} \\ -\frac{3}{5} & \frac{4}{5} & 0 \end{pmatrix} \Rightarrow |\hat{C}| = \begin{pmatrix} 0 & 0 & 0 \\ +\frac{1}{3}\cdot\gamma_1 & 0 & +\frac{2}{3} \\ +\frac{3}{5}\cdot\gamma_1 & \frac{4}{5}\cdot\gamma_2 & 0 \end{pmatrix}$ $\gamma_1 = 0 < 1$ ✓ $\gamma_2 = \frac{2}{3} < 1$ ✓ $\gamma_3 = \frac{8}{15} < 1$ ✓

∴ o critério de Sassenfeld é satisfeito

$\Rightarrow$ o método de Gauss-Seidel converge $\forall x^{(0)}$

para x^* com $Ax^* = b$

Não podemos afirmar nada baseado neste critério sobre a convergência do método de Gauss-Jacobi

(b) Seja $x^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbb{Z}^3$

Temos $\|\hat{A}x^{(0)} - \hat{b}\|_\infty = \left\| \begin{pmatrix} 5 \\ 6 \\ -4 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \\ -3 \end{pmatrix} \right\|_\infty = 1 \leq 1$

$\hat{B} = \begin{pmatrix} 1 \\ 2 \\ \frac{3}{5} \end{pmatrix} \Rightarrow x_1^{(1)} = (0 \ \ 0 \ \ 0)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 1 = 1$

$x_2^{(1)} = \left(-\frac{1}{3} \ \ 0 \ \ -\frac{2}{3}\right)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 2 = 1$

$x_3^{(1)} = \left(-\frac{3}{5} \ \ \frac{4}{5} \ \ 0\right)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \frac{3}{5} = \frac{4}{5} = 0{,}8$

$\therefore x^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 0{,}8 \end{pmatrix} \Rightarrow \|x^{(1)} - x^{(0)}\|_\infty = 0{,}2$

$d_r(x^{(1)}, x^{(0)}) = \frac{0{,}2}{1} = 0{,}2 > \varepsilon$