Inspirado no

Bolzano-Weierstrass Rap:

https://www.youtube.com/watch?v=eM3S74kchoM

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escrevi

The Bisection Rap

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Come on all you students if you wanna be free

from stupid thoughts, then you listen to me

Let f be continuous in [a, b],

sign(f(a)) - sign(f(b))

So we know that f has at least one root

and we're thinking to ourselves "This is really goo....d!"

but how to find a root is what we ask

so let's go ahead and deal with this task

To your right is b and to your left is a

and in between a sequence tries to wind its way

an infinity of x_k's this interval has in it

still you don't know where to find the limit

so you're standing in the middle halfway in between

(a+b)/2 if you know what I mean

to your left a segment half the big one's size

to your right another the same way lies

at least one root lies in the other or the one

you'll never believe what this division has done

if a root lies here,

it's on the left or the right, that's clear!

You slide to a side where f changes sign

x_1's the center of 'n interval half the siz

this interval is half as long,

has a zero if my logic ain't wrong

now you do it again, divide the line in two,

and if you paid attention, you know what to do

you do it k times, things are really gettin' small

(1/2)^k is your interval

with this little space, this little bound

a root of f can still be found

You do this forever until Tisha B'Av

'cause infinite recursion is what we love

a chain o' nested intervals, each inside the last

like lil' Russian dolls gettin' smaller fast

You have to believe and then we're nearly done,

b_k - a_k will be less than epsilon.

Then one often stops

or modulus f(x_k) below epsilon drops.

Generating intervals on and on,

only one point is in every one,

'cause all the left endpoints they have a supremum

the same way the right ones have an infimum

this sup and this inf, they lie in each of these sets

though the distance that's between them's as small as it gets

They are both the same, so I say "What the hell!

I guess it's a limit, I call it L!"

Interval k and all the points that it spans,

I only need one, that's how bad I am

x_k's my name, it's in interval k

quite close to L, I planned it that way!

Pick epsilon as small as you want it to be

nested intervals are working for me

I'll come back with an M so big it'll do:

b - a divided eps' the log base 2

The thing with that M, I picked it so good,

after M x_k's in L's hood

L's hood's eps'lon sized, all intervals lie in it

QED, we have a sequence converging to that limit

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