An Apology for "A Mathematician's Apology" by G. H. Hardy

> *"... modern science ... has recognised the anthropomorphic origin and nature of human knowledge. ... it has recognised that man is the measure of all things, and that there is no other measure."*
>
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Among the greatest scientists in the world, there are many who consider science, from an idealistic point of view, as an *anonymous* human achievement, hence, something that mankind should be proud of (contrasting by the way, with many other shames of our species!). This is the point of view which I want to adopt in this article: consider science in a classic and pure context, independent of industrial technology or military motivations. Following this idea, science is evolving naturally as a consequence of the curiosity of the mind on finding out how things in the world around us work, and how they behave: Is it possible that there are general laws? Are there some principles governing the apparent chaos? Is it possible that the apple which falls is part of the same natural phenomenon as the Moon which rotates around the Earth but does not fall?

The search for such kind of answers is rather exciting for these intellects. Two hundred years ago, Laplace explained this exploration in his impressive *The System of the World*: "One of the strongest passions is the love of truth, in a man of genius" [9, Book V, Chap. IV]. Real scientists do not compete with each other: the challenge is rather to overcome the limitations and ignorance of human beings. Inarguably, some scientific contributions carry more weight than others. But, once scientists view their research as part of an exhilarating scientific voyage, there is no room left for dichotomous attitudes, classifying people as winners or losers. And here starts my criticism of the well-known book of G. H. Hardy (1877-1947): "A Mathematician's Apology". In this book, published for the first time in 1940, he gives his opinion on the mathematical world in twenty-nine short chapters. More recent editions are easier to find and include a foreword by C. P. Snow [5]. Along this article I am going to point out some ideas presented in his book which sound to me either controversial, having prejudice or could be argued in a more respectful and deferential way.

Right from the start, he apologizes for his own criticism, claiming that:

"exposition, criticism, appreciation, is work for second-rate minds." It is a rather surprising beginning (what does he imply about the author of the *Exposition du Systême du Monde* [9], the masterpiece mentioned above?) We all know about very talented philosophers, critics, writers, artists, even journalists who had been and are still playing a fundamental role in the development of science, arts and humanism in general. So, this comment sounds very pretentious coming from a mathematician.

Still in the first chapter he disdains the speech of Alfred E. Housman (1859-1936), Kennedy Professor of Latin in the University of Cambridge, in his Leslie Stephen lecture on the 9th of May 1933: *The Name and Nature of Poetry* when, at the very beginning [6, p. 2] he modestly referred to his previous speech years before in the same Senate-House:

> "In these twenty-two years I have improved in some respects and deteriorated in others, but I have not so much improved as to become a literary critic, nor so much deteriorated as to fancy that I have become one."

He was reinforcing what he had said in 1911 in the Cambridge Inaugural Lecture *The Confines of Criticism,* about literary criticism [7, p. 27]. Concerning this quotation, Hardy declares: "... deplorable that a great scholar and a fine poet should write like this." I apologise for wishing to express exactly the same words about Hardy's declaration.

My disappointment arises specifically from the fact that the book was written by such a great mathematician, who not only left many contributions of his own, but also, the unique occidental mathematician who was considerate enough to recognise the talent of Ramanujan. A quite unusual attitude for that time: help and support bringing to light exceptionally talented people who come from not so (scientifically) prestigious places. For those who are interested in knowing more about the relation between Hardy and Ramanujan, see e.g. the Ranganathan's book [11], where one finds details of Ramanujan's meteoric and short carrier, and his depressive and unhealthy life.

Due to the atrocities of the First World War, Hardy had reasons to condemn the application of science in military matters, in particular, to reprobate the fact that some research on applied mathematics was supporting directly those purposes. Needless to say that applied mathematics is much wider than those military purposes (by the way Bertrand Russell knew that, and focused his pacifism in a more directed way, up to the point of being imprisoned for pacifist acts during the war). For some reason related to this, Hardy was very proud for being a *pure* mathematician ("a real mathematician ... the purest of the pure" as C. P. Snow described in the Foreword), I would say, almost to the point of treating applied mathematics with prejudice. Nevertheless, ironically, contrasting with this stereotype, he also became famous due to a beautiful result on applied mathematics. In 1908 he sent a 2-page letter to the editor of the *Science* [4] with results (concurrently with the German physician W. Weinberg) on how proportions of dominant and recessive genetic traits propagate in a large mixed population, the well-known Hardy-Weinberg law. This result became centrally important in many population genetic problems including hemolytic disease, see e.g., among many others introductory textbooks on the subject, Spiess [12]. Concerning Hardy's outstanding legacy on pure mathematics, his contributions are mainly on theory of Diophantine analysis, divergent series, Fourier series, Riemann zeta-function and the distributions of primes. His greatest collaborators were Littlewood and Ramanujan.

Back to the book which is the focus of my criticism, in Chapter seven he describes his ideas on motivation for scientific research. He emphasizes some motives which go in some sense against those ideas I presented in the first two paragraphs. He claims that, besides intellectual curiosity, the inspirations come from professional pride, "ambition, desire for reputation, and the position, even the power or the money, which it brings". I agree that these latter points represent part of motivation for many among us. But definitely they also represent delicate points which, when overcharged can induce acts which hurt (or could be in the borderline of) ethic. Regrettably, he emphasizes the last few motives:

> "So if a mathematician, or a chemist, or even a physiologist, were to tell me that the driving force in his work had been the desire to benefit humanity, then I should not believe him (nor should I think the better of him if I did)".

This phrase could sound as an offence for those working on science for idealism or for those really working for "love of truth" or to "benefit the humanity", and we know that these people, though the minority, do exist!

These élitist or competitive ideas, in the sense that the only contributions which matter are made by those among the best, is pejoratively reinforced along the book. They are expressed in words like: "mathematical fame ... one of the soundest and steadiest investment" (Chap. 8) ,"second-rate mind" or "... have done something beyond the powers of the vast majority of men" (Chap. 6).

Philosophically speaking, I think nowadays it is hard to find somebody who agrees with his statement in Chapter 27 concerning a phrase of Hogben: "The mathematics which can be used 'for ordinary purposes by ordinary men' is negligible." Firstly, on what concerns the élitist aspect of this disdainful phrase, compare it with the Preface of *The Mathematics of Great Amateurs* [1], where Coolidge says that, in his book: "... the number of men included could easily be doubled or trebled". Secondly, on what concerns the presumptuous aspect of Hardy's phrase, contrast it with the modest and respectful declaration of Laplace, as simple and deep as that:

> "... the ingenious method of expressing all numbers by means of ten symbols, each symbol receiving a value of position as well as an absolute value; a profound and important idea which appears so simple to us now that we ignore its true merit."

This quotation of Laplace opens the second chapter of the remarkable book *Number: the Language of Science* by Tobias Dantzig [2], author of the epigraph at the top of this article. His book received many compliments of 20th century top scholars, including Albert Einstein. Many other authors also treat the subject differently from Hardy's approach. Besides the already mentioned Laplace book [9], it is hard to prevent to mention the work of some other illustrious authors (I apologise in advance for omitting so many of them in this short article) like Morris Kline, in particular his comprehensive survey *Mathematical Thought from Ancient to Modern Times* [8], Lancelot T. Hogben (who, in Chapter 27 and 28 Hardy makes clear that he belongs to a different school from his own), some contemporaries like David Fowler, Carl Boyer, or Ian Stewart with his vast work involving updated mathematical objects. It is equally interesting to note some introductory texts regarding philosophy of science in general, like (back to the 17th century) the classic "dialogue" written by the founder of the modern physics Galileo Galilei [3], some of H. Poincaré's works ([10], e.g.) and the more philosophical works of Bertrand Russel and Raymond Wilder. Many of these books go in an orthogonal direction of the ideas presented by Hardy.

We all know that there are many books on history and philosophy of mathematics which are partial, elitists or even tendentious in many aspects. But what really surprises me is the gap between Hardy's attitude in his life and the ideas expressed in his book. Naturally, people develop different concepts, sensitiveness and points of view along their intellectual carrier. Nevertheless, Hardy's book, compared to others, humanistically speaking, reflects a very dry, bitter and thorny philosophy.

Finally, for those who are interested in getting to know more about the kind of "irony" Hardy enjoyed, have a look in the note by A. M. Vershik [13]. Without wanting to dislodge Hardy's book from its established status as a statement on mathematical philosophy by a thoughtful and articulate mathematician, I recommend that in reading it we ask ourselves whether some of the ideas presented are presumptuous and scornful to the point of hurting the development of science and humanism in general.

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