Topics in Ergodic Theory and Dynamical Systems
MPI-MiS – winter sem. 2012/2013

1 - Invariant measures and recurrence

1.1 - Invariant measures
1.2 - Poincaré Recurrence Theorem
  1.2.1 - Measure Theoretical version
  1.2.2 - Kac's Lemma
  1.2.3 - Poincaré Recurrence Theorem II - Topological version
1.3 - Examples
1.4 - Existence of invariant measures for continuous transformations

2 - Ergodicity

2.1 – Ergodic Theorems
  2.1.1 - Birkhoff Ergodic Theorem
  2.1.2 - Von Neumann Ergodic Theorem
  2.1.3 - Subadditive Ergodic Theorem
2.2 – Ergodicity

3 - Examples

3.1 - Examples of ergodic theory
  3.1.1 - Topological Representatives of Ergodic Systems
  3.1.2 - Shifts and their Topological and Ergodic Properties
  3.1.3 - Examples from Combinatorics and Number Theory
3.2 - Furstenberg's school
  3.2.1 - Multiple Recurrence and its Application to Prime Progressions
  3.2.2 - Ideas and Examples of the Proof of Multiple Recurrence

4 - Topology and dynamics I

4.1- Oseledec's Multiplicative Ergodic Theorem
4.2- Lyapunov Exponents (I)

5 - Topology and dynamics II

5.1- Lyapunov Exponents (II)
5.2- Pesin's stable manifold theory
5.3- Katok's closing and shadowing lemma for hyperbolic measures

6 - Thermodynamics formalism

7 - Physical or SRB measures

8 - Entropy

8.1 - Topological Entropy
8.1.1 - Definition of Adler-Konheim-McAndrew and Bowen
8.1.2 - Properties

8.2 - Kolmogoroff-Sinai Entropy
8.2.1 - Definition and some properties
8.2.2 - Katok's KS-Entropy Definition a la Bowen

8.3 - Relations and Properties of the Entropies
8.3.1 - (Local) Variational Lemmas
8.3.2 - Existence of Bernoulli subsystem
8.3.3 - Ergodic Equivalence of Bernoulli shifts

9 - Chaos of dynamical Systems

9.1 - Sensitivity
9.1.1 - Auslander-Yorke dichotomy and Deveney's Chaos
9.1.2 - Chaos via Hyperbolicity and Expensiveness
9.2 - Statistical and Topological Chaos
9.2.1 - Furstenberg's "statistical chaos"
9.2.2 - Li-Yorke Scrambled Sets
9.2.3 - Entropy Chaos implies Li-Yorke Chaos

10 - One-dymensional dynamics