Holomorphic dynamics, polynomials, polynomial-like maps and parabolic-like mappings

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Let $z \in \widehat{C}$, and let f be a holomorphic map on $\widehat{\mathbb{C}}$, the *orbit* of z under f is the sequence $\{z, f(z), f^2(z), ...\}$ (where f^n means f composed to itself n-times). The main activity in holomorphic dynamics is the study of the asymptotic behaviour of such orbits and the resulting classification of points in $\widehat{\mathbb{C}}$. The *Fatou set* is the set of points z such that the family (f^n) is equicontinuous near z; the dynamics is chaotic on the complementary *Julia set*. An important special case is given by polynomial maps of $\widehat{\mathbb{C}}$. In the polynomial case the Julia set of a map f is the boundary of the basin of the (super) attracting fixed point at infinity. In this situation is useful to define the *filled Julia set* to be the complement of the basin of attraction of infinity (see, for example, [M]).

In 1985, Adrien Douady and John Hamal Hubbard published a groundbreaking paper entitled *On the dynamics of polynomial-like mappings* ([DH]). A polynomial- like mapping is a proper holomorphic map $f : U' \to U$, where $U', U \approx \mathbb{D}$, and $U' \subset\subset U$. This definition captures the behaviour of a polynomial in a neighbourhood of its filled Julia set. A polynomial-like map of degree d is determined up to holomorphic conjugacy by its internal and external classes, that is, the (conjugacy classes of) the restrictions to the filled Julia set and its complement. In particular the external class is a degree d real-analytic orientation preserving and strictly expanding self-covering of the unit circle: the expansivity of such a circle map implies that all the periodic points are repelling, and in particular not parabolic.

We extended the polynomial-like theory to a class of parabolic mappings which we called parabolic-like mappings. A parabolic-like mapping is an object similar to a polynomial-like mapping, but with a parabolic external class; that is to say, the external map has a parabolic fixed point, whence the domain is not contained in the codomain ([L1] and [L2]).

In these talks we will give an overview over 1-dim complex dynamics, with particular interest on the dynamics of (quadratic) polynomials on the Riemann spehere. We will introduce the polynomial-like mapping theory, and finally we will present the parabolic-like mapping theory.

References

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[M] J. Milnor, *Dynamics in One Complex Variable*, Annals of Mathematics Studies, (2006).