Bootstrap Prediction in DCC-GARCH Multivariate Volatility Model with Normal Distribution

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Introduction

In the field of financial time series, there are few works on procedures to obtain prediction intervals (PI) for returns, volatilities and covariances when are in multivariate case. In general, PI is calculated under the assumption that the model is known and has normal distribution errors, however features in financial time series make that the usual approach is not adequate. An alternative to this problem is to obtain PI using bootstrap procedures, which do not require the choice of a distribution to the innovations and can handle the problem of the estimation error. [3] present a method for calculate bootstrap PI in GARCH models, [4] adapt this methodology and calculate bootstrap PI for models EGARCH and GJR-GARCH. Prediction intervals in multivariate volatility models has been understudied. This work is one first approach for obtain bootstrap prediction intervals in multivariate volatility models.

Methodology

[3] proposed a bootstrap algorithm to obtain PI for returns and volatilities in GARCH models. We adapt the algorithm to Multivariate GARCH Model DCC_E [1], [2].

Algorithm

Consider r_T a sequence two dimensional of T observations generated by the process DCC(1,1)-GARCH(1,1). The algorithm is described for process DCC(1,1)-GARCH(1,1) with $K = 2$ dimensions, but it is easily generalized to a DCC(m,n)-GARCH(p,q) process with more than $K = 2$ dimensions in a straightforward way.

- Step 1: Obtain estimates of the process parameters DCC-GARCH(1,1) $\theta = (\omega_1, \omega_2, \alpha_1, \alpha_2, \beta_1, \beta_2, a, b)$ give by $\hat{\theta} = (\hat{\omega_1}, \hat{\omega_2}, \hat{\alpha_1}, \hat{\alpha_2}, \hat{\beta}_1, \hat{\beta}_2, \hat{a}, \hat{b})$. Calculate the vectors of residues centered $\hat{\epsilon_t} - \bar{\hat{\epsilon}}$, where $\hat{\epsilon_t} = D_t^{-1}\mathbf{r_t}$ and D_t is a diagonal matrix with components $D_{kkt} = \hat{\sigma}_{kt}$;
- Step 2: Obtain the residues standardized $\varepsilon_t = \bar{\epsilon_t}\hat{R_t}^{-1/2}$ and denote by $\hat{F}_T$ the empirical distribution of the residues standardized;
- Step 3: Obtain the residues bootstrap $\epsilon_t^* = \varepsilon_t^* \hat{R_t^*}^{1/2}$ $t = 1, ..., T$ where $\hat{R}_t^* = Q_t^{,*-1/2} Q_t^* Q_t^{,*-1/2}$, $Q_t^* = \bar{Q}^*(1 - \hat{a} - \hat{b}) + \hat{a}(t(\epsilon_{t-1}^*)\epsilon_{t-1}^*) + \hat{b}Q_{t-1}^*$, $\bar{Q}^* = Corr(\epsilon)$, $Q_1^* = R_1^* = \bar{Q}^*$, $\epsilon_1^* = \epsilon_1$ and $Q_t^{,*1/2}$ is a diagonal matrix with components $Q_{kkt}^{,*1/2} = \sqrt{q_{kkt}^*}$;
- Step 4: Generate a bootstrap series $r_T^* = (r_{1T}^*, r_{2T}^*)$, where r_{kT}^* is calculated using the following recursion:

$$\begin{aligned} \sigma_{kt}^{*2} &= \hat{\omega}_k + \hat{\alpha}_k r_{k(t-1)}^{*2} + \hat{\beta}_k \sigma_{k(t-1)}^{*2} \\ r_{kt} &= \epsilon_{kt}^* \sigma_{kt}^*, \end{aligned} \tag{1}$$

- Step 5: Compute $Q_T^*(0)$, $R_T^*(0)$ and $H_T^*(0)$. $H_t^* = \sigma_t^* R_t^* \sigma_t^*$ where $R_t^* = Q_t^{,*1/2} Q_t^* Q_t^{,*1/2}$, $Q_t^* = \bar{Q}^* + \hat{a^*}(t(e_{t-1})e_{t-1}) + \hat{b^*}Q_{t-1}^*$, $e_t = r_t D_t^{*-1}$, $\bar{Q}^* = Corr(\hat{\epsilon})$, $R_1^* = \hat{R}_1^*$, $Q_1^* = \hat{Q}_1^*$ and $\sigma_t^* = c(\sigma_{1t}^*, \sigma_{2t}^*)$ and σ_{kt}^* is calculated using the following recursion:

$$\begin{aligned} \sigma_{kT}^{*2} = &\frac{\hat{\omega_k^*}}{1 - \hat{\alpha_k^*} - \hat{\beta_k^*}} \\ &+ \hat{\alpha_k^*} \sum_{j=0}^{T-2} \hat{\beta_k^{*j}} \left(r_{k(T-j-1)} - \frac{\hat{\omega_k^*}}{1 - \hat{\alpha_k^*} - \hat{\beta_k^*}}\right); \end{aligned} \tag{2}$$

- Step 6: Calculate forecasts of returns,volatilities and covariance h steps ahead, $h = 1, 2, ...$ using the following recursion: $r_T^*(h) = \epsilon_T^*(h) D_T^*(h)$ $H_T^*(h) = \sigma_T^*(h) R_T^* \sigma_T^*(h)$, where $\sigma_T^*(h) = c(\sigma_{1T}^*(h), \sigma_{2T}^*(h))$ and is calculated using the following recursion:

$$\begin{aligned} \sigma_{kT}^*(h) &= \hat{\omega_k^*} + \hat{\alpha_k^*} r_{kT}^*(h-1) + \hat{\beta_k^*} \sigma_{kT}^*(h-1), \\ r_{kT}(h) &= \epsilon_T^*(h) \sigma_{kT}^*(h), \end{aligned} \tag{3}$$

$R_T^*(h) = Q_T^{,*1/2}(h) Q_T^*(h) Q_T^{,*1/2}(h)$ $Q_T^*(h) = \bar{Q}^*(1 - \hat{a^*} - \hat{b^8}) + \hat{a^*}(t(\epsilon_T^*(h))\epsilon_T^*(h)) + \hat{b^*}Q_T^*(h-1)$ $\epsilon_T^*(h) = \varepsilon_T^*(h) R_T^{*1/2}(h)$ where $\varepsilon_T^*(h) \sim$ i.i.d $\hat{F}_T$ $\epsilon_T^*(0) = r_T D_T^{*-1/2}$

- Step 7: Repeat steps 4 - 6, B times and there by obtain B bootstrap replicates $(r_T^{*(1)}(h), ..., r_T^{*(B)}(h))$, where $y_T^{*(l)}(h)$ is $r_T^{*(l)}(h)$, $\sigma_T^{*(1)}(l)$ or $Cov_T^{*(1)}(l)$ respectively. The limits of prediction for $r_T(h)$, $\sigma_T(h)$ and $Cov_T(h)$ are defined as the quantiles of bootstrap $r_T^*(h)$, $\sigma_T^*(h)$ e $Cov_T^*(h)$.

Simulation Study

Here we analyze the performance of the suggested algorithm through Monte Carlo simulation. We consider the model DCC-GARCH(1,1) with normal distribution in the disturbances. The model is:

$$\begin{aligned} r_t &= H_t^{1/2} \varepsilon_t \\ H_t &= D_t R_t D_t \end{aligned} \tag{4}$$

where

$$R_t = Q_t^{'-1/2} Q_t Q_t^{'-1/2} \tag{5}$$

$$Q_t = (1 - 0.015 - 0.94)\bar{Q} + 0.015\epsilon_{t-1}\epsilon_{t-1}^T + 0.94Q_{t-1} \tag{6}$$

$$Q_t^{'-1/2} = \begin{bmatrix} \sqrt{q_{11t}} & 0 \\ 0 & \sqrt{q_{22t}} \end{bmatrix}, D_t = \begin{bmatrix} \sigma_{1,t} & 0 \\ 0 & \sigma_{2,t} \end{bmatrix} and$$

$$\sigma_{1,t}^2 = 0.01 + 0.1r_{1,t-1}^2 + 0.86\sigma_{1,t-1}^2 \tag{7}$$

$$\sigma_{2,t}^2 = 0.005 + 0.07r_{2,t-1}^2 + 0.88\sigma_{2,t-1}^2 \tag{8}$$

The persistence in the two models GARCH(1,1) are 0.96 and 0.95 respectively.

Simulation Algorithm

For the model we ran 100 replications. For each replication we have three steps. Thus for the k-th replication have:

- Step 1: Generate a two variate series with size $T = 1000$. That is generate r_t and σ_t for $t = 1, \cdots, 1000$;
- Step 2: Run steps 1 to 7 given in before subsection to find the $h = 1, 2, 3, 4, 5$ steps-ahead 95%-PI for the returns, volatilities and covariance of the two series;
- Step 3: Generate 1000 sets of future values $\{r_{k,T+j}^i, j = 1, \cdots, 5, i = 1, \cdots, 1000\}$, $\{\sigma_{k,T+j}^i, j = 1, \cdots, 5, i = 1, \cdots, 1000\}$ and $\{Cov_{k,T+j}^i, j = 1, \cdots, 5, i = 1, \cdots, 1000\}$ considering that the previous observations are given in Step 1. For $h = 1, 2, 3, 4, 5$, denote by $p_q^r(h)$ the proportions of the values $\{r_{k,T+h}^i, i = 1, \cdots, 1000\}$ which are inside, below the lower and above the upper limit of the PI found in step 2. Define similarly $p_q^\sigma(h)$ and $p_q^{Cov}(h)$ for volatilities and the covariance respectively.

Results

Table 1: Summary of the Simulations of the prediction intervals for returns and volatilities of DCC-GARCH(1,1) models. h-steps-ahead prediction. Nominal coverage 95%.

	series 1				series 2			
Horizon	Average coverage	Standard error	Av. below coverage	Average length	Average coverage	Standard error	Av. below coverage	Average length
returns								
h = 1	0.9476	0.0134	0.0256	1.8586	0.9477	0.0175	0.0255	1.2000
h = 2	0.9488	0.0136	0.0261	1.8716	0.9478	0.0158	0.0246	1.2058
h = 3	0.9492	0.0134	0.0251	1.8854	0.9468	0.0147	0.0255	1.2067
h = 4	0.9489	0.0136	0.0244	1.8806	0.9446	0.0149	0.0269	1.2005
h = 5	0.9483	0.0141	0.0255	1.8902	0.9474	0.0147	0.0252	1.2121
volatilities								
h = 1	0.9300	0.2564	0.0200	0.0235	0.9300	0.2564	0.0400	0.0449
h = 2	0.9555	0.1063	0.0243	0.0398	0.9397	0.1302	0.0290	0.0752
h = 3	0.9452	0.0952	0.0318	0.0479	0.9392	0.1092	0.0291	0.0921
h = 4	0.9388	0.0947	0.0376	0.0541	0.9364	0.0953	0.0315	0.1042
h = 5	0.9350	0.0921	0.0396	0.0582	0.9310	0.0920	0.0343	0.1135

Table 2: Summary of the Simulations of the prediction intervals for covariances of DCC-GARCH(1,1) models. h-steps-ahead prediction. Nominal coverage 95%.

Horizon	Average coverage	Standard error	Av. below coverage	Av. above coverage	Average length
h = 1	0.9300	0.2564	0.0200	0.0500	0.0235
h = 2	0.9555	0.1063	0.0243	0.0202	0.0398
h = 3	0.9452	0.0952	0.0318	0.0230	0.0479
h = 4	0.9388	0.0947	0.0376	0.0236	0.0541
h = 5	0.9350	0.0921	0.0396	0.0254	0.0582

Application VaR

The bootstrap VaR 95% was calculated considering that the portfolio was built considering the same weighing to each asset. We ran 100 replications, thus for each replication we have:

- Obtain $\{r_T^{*i}(h),\ h = 1, \cdots, 5,\ i = 1, \cdots, 1000\}$
- Compute the empirical distribution bootstrap of the log-return h steps ahead of the portfolio $log(\sum_{k=1}^{K} p_k e^{\sum_{j=1}^{h} r_{kT}^{i*}(j)})$ where $\sum_{k=1}^{K} p_k = 1$
- The $VaR_{boot}(h)$ 95% is obtain how the quantile 5% of the log-return of the portfolio

Results VaR

For each replication the $VaR_{boot}(h)$ 95% was compared with the log-return of the portfolio h steps ahead obtained by simulation. Is the value of the log-return of the portfolio h steps ahead was less than $VaR_{boot}(h)$ 95% then the value 1 was assigned otherwise the value 0 was assigned. The results are in the Table 3.

Table 3: Summary of Value-at-Risk 95% h-steps-ahead using bootstrap procedure.

	VaR 95%				
Steps ahead	h = 1	h = 2	h = 3	h = 4	h = 5
Mean	0.0512	0.0503	0.0513	0.0512	0.0508
D.P	0.0125	0.0115	0.0109	0.0106	0.0111

References

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QR code