Resolução P3

1) $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ yze^{xz} & (e^{xz}+2y) & (xye^{xz}+e^{z}) \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ \partial_x & \partial_y \\ yze^{xz} & (e^{xz}+2y) \end{matrix}$ (0,25)

$= (xe^{xz} - xe^{xz})\hat{i} + (ye^{xz} + xyze^{xz} - ye^{xz} - xyze^{xz})\hat{j} + (ze^{xz} - ze^{xz})\hat{k}$

$= 0$

Encontrando o potencial:

i) $\dfrac{\partial f}{\partial x} = yze^{xz} \Rightarrow f = ye^{xz} + g(y,z)$

ii) $\dfrac{\partial f}{\partial y} = e^{xz} + 2y \Rightarrow \cancel{e^{xz}} + \dfrac{\partial g}{\partial y} = \cancel{e^{xz}} + 2y \Rightarrow g = y^2 + h(z)$

iii) $\dfrac{\partial f}{\partial z} = (xye^{xz} + e^{z}) \Rightarrow \cancel{xye^{xz}} + h'(z) = \cancel{xye^{xz}} + e^{z}$

$\Rightarrow h(z) = e^{z} + C$

$\therefore \; f(x,y,z) = ye^{xz} + y^2 + e^{z} + C$ (1,25)

Conferindo: $\vec{\nabla} f = (yze^{xz},\; e^{xz} + 2y,\; xye^{xz} + e^{z}) = \vec{F}$ ✓

Note que $\vec{r}(2) = (2^2+1,\, 2^2-1,\, 2^2-2.2) = (5,3,0)$

$\vec{r}(0) = (0^2+1,\, 0^2-1,\, 0^2-2.0) = (1,-1,0)$

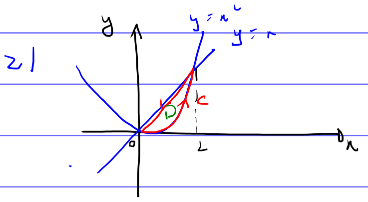
$\therefore \displaystyle\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(2)) - f(\vec{r}(0)) = f(5,3,0) - f(1,-1,0)$

$= 3e^{5\cdot 0} + 3^2 + e^0 + C - (-e^{1\cdot 0} + (-1)^2 + e^0 + C)$

$= 3 + 9 + 1 + C + 1 - 1 - 1 - C$

$= \boxed{12}$ (0,5)

2)



$\displaystyle\oint_C \vec{F}\cdot d\vec{r} = \iint_D \left(\frac{\partial Q}{\partial y} - \frac{\partial P}{\partial x}\right) dA$

$= \displaystyle\int_0^1 \int_{x^2}^{x} (3 - 2y)\, dy\, dx$ (1,0)

$= \displaystyle\int_0^1 \left[3(x - x^2) - (x^2 - x^4)\right] dy$

$= \displaystyle\int_0^1 (3x - 4x^2 + x^4)\, dx$

$= \dfrac{3x^2}{2}\Big|_0^1 - \dfrac{4x^3}{3}\Big|_0^1 + \dfrac{x^5}{5}\Big|_0^1$

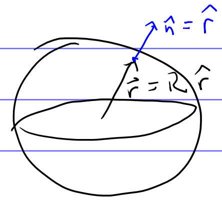
$= \dfrac{3}{2} - \dfrac{4}{3} + \dfrac{1}{5}$

$= \dfrac{+45 - 40 + 6}{30} = \boxed{\dfrac{11}{30}}$ (0,5)

O Campo não é conservativo, pois $\oint_C \vec{F}\cdot d\vec{r} \neq 0$ (0,5)

3) Pelo Teorema do Divergente

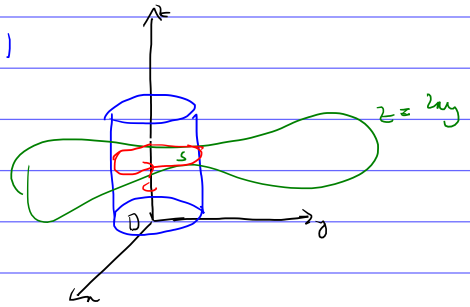
$$\frac{1}{3}\iint_S \vec{r}\cdot\hat{n}\,dS = \frac{1}{3}\iiint_V (\vec{\nabla}\cdot\vec{r})\,dV = \frac{1}{3}\iiint 3\,dV = \iiint_B dV = \text{volume de } B$$ (1,0)



$$\therefore \frac{1}{3}\iint_S \vec{r}\cdot\hat{n}\,dS = \frac{1}{3}\int_0^{2\pi}\int_0^{\pi}(R\hat{r}\cdot\hat{r})(R^2\sin\varphi\,d\varphi\,d\theta)$$

$$= \frac{R^3}{3}\int_0^{2\pi}\int_0^{\pi}\sin\varphi\,d\varphi\,d\theta = \boxed{\frac{4\pi R^3}{3}}$$ (1,0)

4)



p/ superfície S:

$\vec{r} = (x, y, 2xy)$

$\vec{r}_x = (1, 0, 2y)$

$\vec{r}_y = (0, 1, 2x)$

$$\vec{r}_x \times \vec{r}_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2y \\ 0 & 1 & 2x \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 0 \\ 0 & 1 \end{matrix}$$

$$= -2y\,\hat{i} - 2x\,\hat{j} + 1\,\hat{k}$$

$$\therefore \|\vec{r}_x \times \vec{r}_y\| = \sqrt{1+4(x^2+y^2)}$$

$$\therefore \hat{n} = \frac{-2y\,\hat{i} - 2x\,\hat{j} + \hat{k}}{\sqrt{1+4(x^2+y^2)}}$$ (0,75)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ e^x\cos y & x & -xy \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ \partial_x & \partial_y \\ e^x\cos y & x \end{matrix} = -x\,\hat{i} + y\,\hat{j} + \hat{k}$$

$$\therefore \vec{\nabla}\times\vec{F}\cdot\hat{n}\,dS = (-x\,\hat{i} + y\,\hat{j} + \hat{k})\cdot\frac{(-2y\,\hat{i} - 2x\,\hat{j} + \hat{k})}{\sqrt{1+4(x^2+y^2)}}\sqrt{1+4(x^2+y^2)}\,dA$$ (0,75)

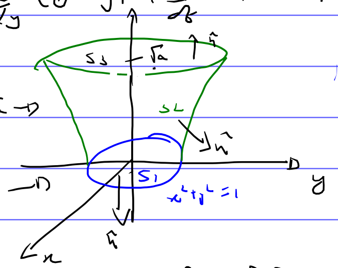
$$= 2xy - 2xy + 1$$

$$\therefore \oint_C \vec{F}\cdot d\vec{r} = \iint_S \vec{\nabla}\times\vec{F}\cdot\hat{n}\,dS = \iint_D dA = \int_0^{\pi}\int_0^1 r\,dr\,d\theta = \boxed{\pi}$$ (0,5)

5) a) $$\vec{G} = \vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ \frac{z}{2}y + ze^x & \frac{z}{2}x + ze^y & xy \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ \partial_x & \partial_y \\ \frac{z}{2}y + ze^x & \frac{z}{2}x + ze^y \end{matrix}$$

$$= (x - e^y)\,\hat{i} + (e^x - y)\,\hat{j} + \left(\frac{z}{2} + \frac{z}{2}\right)\hat{k}$$

$$\therefore \vec{\nabla}\cdot\vec{G} = \frac{\partial}{\partial x}(x - e^y) + \frac{\partial}{\partial y}(e^x - y) + \frac{\partial}{\partial z}(a) = 1 - 1 + 0 = \boxed{0}$$ (0,5)



b) S_1: $z = \sqrt{x^2+y^2-1}$, $0 \le z \le \sqrt{a} \to$

S_2: $x^2+y^2 \le 1$, $z = 0$

Pelo Teorema do Divergente $$\iint_{\tilde{S}} \vec{G}\cdot\hat{n}\,dS = \iiint \vec{\nabla}\cdot\vec{G}\,dV = 0$$

(com $\tilde{S} = S_1 \cup S_2 \cup S_3$, onde

$S_3 = x^2+y^2 \le 1+a$, $z = \sqrt{a}$

$$\iint_S \vec{G}\cdot\hat{n}\,dS + \iint_{S_3} \vec{G}\cdot\hat{n}\,dS = 0$$ (0,5)

$$\Rightarrow -6\pi = \iint_S \vec{G}\cdot\hat{n}\,dS = -\iint_{S_3} \vec{G}\cdot\hat{n}\,dS = -\iint_{S_3} (x-e^y, e^x-y, a)\cdot(0,0,1)\,dS$$

$$= -a\iint_{S_3} dS = -a\pi(1+\omega)$$

$$\therefore +6 = a(1+\omega) \Rightarrow \boxed{a=2}$$ (1, 0)