Second Workshop on Nonlinear Dispersive Equations

Celebrating the 70th Anniversiry of Marcia Scialom

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Abstracts - Conference Talks

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(1) John Albert, University of Oklahoma, USA, *jalbert@ou.edu*

Title: On the well-posedness of the Cauchy problem for the dispersion-managed nonlinear Schrödinger equation

Abstract: The dispersion-managed nonlinear Schrödinger (DMNLS) equation is a model equation, first derived by Gabitov and Turitsyn, for the propagation of light pulses through an optical fiber made by gluing together lengths of glass with alternately positive and negative dispersive properties. The equation takes the form of a nonlinear Schrodinger equation in which the nonlinearity is replaced by a nonlocal averaging term, and the dispersive term is multiplied by a coefficient α which represents the average value of the dispersion. In actual practice, the fiber is usually designed so that α is zero, or nearly zero.

Somewhat surprisingly, Kunze has shown that even when α is zero, so that there is no net dispersion, the equation admits ground-state solitary-wave solutions, due to the dispersive properties of the nonlinear term. In this talk we review Kunze's result, along with recent improvements due to Hundertmark et al., and discuss the well-posedness of the equation for initial data in L^2, which provides the natural setting for Kunze's result.

Joint work with Estapraq Kahlil.

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(2) Miguel Alejo, UFSC, Brazil, *miguel.alejo@ufsc.br*

Title: Stability of mKdV Breathers in the Energy Space

Abstract: In this talk I will show some recent results about the H^1 stability of breather solutions of mKdV. I will also present B"acklund transformations for the mKdV and I will explain how we use them as a technical tool to get stability at the level of H^1 regularity.

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(3) Jaime Angulo, IME-USP, Brazil, *angulo@ime.usp.br*

Title: Extension theory of symmetric operators: a new approach in stability of standing waves for NLS equation with point interactions

Abstract: The aim of this talk is to demonstrate effectiveness of extension theory of symmetric operators for investigation of stability of standing waves for semi-linear Schrödinger equation with δ- and δ'-interaction. Our approach relies on the theory of stability for Hamiltonian systems which are invariant under a one-parameter group of operators.

Joint work with N. Goloshchapova, Department of Mathematics, IME-USP.

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(4) Jerry Bona, UIC, Chicago, USA, *jbona@uic.edu*

Title: Blood Flow and Waves on Trees

Abstract: A crude model of arterial circulation is introduced and studied. The model involves a system of nonlinear wave equations coupled through boundary consistency conditions.

After a brief indication of the derivation of the model, interest is turned to whether or not it makes any sense. That is to say, is it well posed with the natural auxiliary data pertaining to blood flow? Addressing this issue will comprise the remainder of the lecture.

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(5) Xavier Carvajal , UFRJ, Brazil, *carvajal@im.ufrj.br*

Title: Blow-up and growth of norms to a system of coupled Schrödinger equations

Abstract: This paper is concerned with the behaviour of solutions to a system of coupled Schrödinger equations

$$\begin{cases} iu_t + \Delta u + (\alpha|u|^{2p} + \beta|u|^q|v|^{q+2})u = 0, \\ iv_t + \Delta v + (\alpha|v|^{2p} + \beta|v|^q|u|^{q+2})v = 0, \\ u(x,0) = u_0(x), \qquad v(x,0) = v_0(x), \end{cases}$$

which has applications in many physical problems, especially in nonlinear optics. In particular when the solution there exists globally we obtain the growth of the solutions in the energy space. Also we obtain some conditions in order to obtain blow-up in this space.

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(6) Adan Corcho, IM-UFRJ, Brazil, *adan.corcho@gmail.com*

Title: Blow-up results for an energy critical perturbation of the cubic non-linear Schrödinger equation

Abstract: We consider the Schrödinger-Debye system in $\mathbb{R}^n$ for dimensions $n = 3, 4$. Developing on previously known local well-posedness results, we start by establishing global well-posedness in $H^1(\mathbb{R}^3) \times L^2(\mathbb{R}^3)$ for a "large" class of initial data. We then concentrate on the initial value problem in $n = 4$, which is the energy-critical dimension for the corresponding cubic nonlinear Schrödinger equation. We start by proving local well-posedness in $H^1(\mathbb{R}^4) \times H^1(\mathbb{R}^4)$. Then, for the focusing case of the system, we derive a virial type identity and finally use it to prove that solutions cannot exist for all times, by carefully controlling the non-linear terms from the Debye relaxation in order to employ a standard convexity argument.

This work is in collaboration with J. Drumond Silva (IST, Lisbon)

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(7) Amin Esfahani, Damghan University, Iran, *saesfahani@gmail.com*

Title: Sharp constant of an anisotropic Gagliardo-Nirenberg-type inequality and applications

Abstract: In this paper we establish the best constant of an anisotropic Gagliardo-Nirenberg-type inequality related to the Benjamin-Ono-Zakharov-Kuznetsov equation. As an application of our results, we prove the uniform bound of solutions for such a equation in the energy space.

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(8) Luca Fanelli, SAPIENZA Università di Roma, Italy, *fanelli@mat.uniroma1.it*

Title: Improved time decay for magnetic Schrödinger evolutions

Abstract: We will show some recent results about the relation between Laplace-Beltrami operators of the form

$$L_{A,a} = (i\nabla_{\mathbb{S}^{d-1}} + A(\theta))^2 + a(\theta), \qquad A : \mathbb{S}^{d-1} \to \mathbb{R}^d, \quad a : \mathbb{S}^{d-1} \to \mathbb{R}$$

and dispersive properties of electromagnetic Schrödinger flows e^{itH}, with

$$H = H_{A,a} = (i\nabla + |x|^{-1}A(\theta))^2 + |x|^{-2}a(\theta).$$

This family includes relevant examples, as the Aharonov-Bohm potential and the point-dipole. Notice that H is a critical perturbation of $H_{0,0}$ with respect to the scale of H^1, by the Hardy inequality. We will show a mechanism connecting spectral properties of $L_{A,a}$ to time-dispersive properties of e^{itH}. In addition, we will prove some quantitative diamagnetic improvement of the free decay, in suitable topologies.

The results are obtained in collaboration with V. Felli (Milano-Bicocca), M. Fontelos (Madrid - ICMAT), G. Grillo (Milano - Politecnico), H. Kovařík (Brescia - Universit), A. Primo (Madrid - UAM).

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(9) Luiz Gustavo Farah, UFMG, Brazil, *lgfarah@gmail.com*

Title: On the lack of compactness and existence of maximizers for some Airy-Strichartz inequalities

Abstract: Recently, in a joint work with Ademir Pastor [1], we give a simple proof of the classical Kenig, Ponce and Vega well-posedness result for the generalized KdV equation [2]

$$\begin{cases} \partial_t u + \partial_x^3 u + \partial_x(u^{k+1}) = 0, \quad x \in \mathbb{R}, \quad t > 0, \quad k \geq 4, \\ u(x,0) = u_0(x). \end{cases}$$

The key ingredient in the proof is the following Airy-Strichartz estimate

$$\|U(t)u_0\|_{L_x^{5k/4} L_t^{5k/2}} \leq C_k \|u_0\|_{\dot{H}_x^{s_k}},$$

where $k > 4$, $s_k = (k-4)/2k$ and $U(t)$ denotes the linear propagator for the KdV equation.

Our goal here is to prove the existence of maximizers for the above inequality. The main tool we use is a linear profile decomposition for the Airy equation with initial data in $\dot{H}_x^{s_k}(\mathbb{R})$. As a consequence, we also establish the existence of maximizers for a more general class of Strichartz type inequalities associated to the Airy equation.

This is a joint work with Henrique Versieux (UFRJ). The author was partially supported by CNPq/Brazil and FAPEMIG/Brazil.

References

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(10) Axel Grünrock, Universität Düsseldorf, Germany, *gruenroc@math.uni-duesseldorf.de*

Title: On the generalized Zakharov-Kuznetsov equation at critical regularity

Abstract: The Cauchy problem for the generalized Zakharov-Kuznetsov equation

$$\partial_t u + \partial_x \Delta u = \partial_x u^{k+1}, \qquad u(0) = u_0$$

is considered in space dimensions $n = 2$ and $n = 3$ for integer exponents $k \geq 3$. For data $u_0 \in \dot{B}^{s_c}_{2,q}$, where $1 \leq q \leq \infty$ and $s_c = \frac{n}{2} - \frac{2}{k}$ is the critical Sobolev regularity, it is shown, that this problem is locally well-posed and globally well-posed, if the data are sufficiently small. The proof follows ideas of Kenig, Ponce, and Vega and uses estimates for the corresponding linear equation, such as local smoothing effect, Strichartz estimates, and maximal function inequalities. These are inserted into the framework of the function spaces U^p and V^p introduced by Koch and Tataru.

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(11) Rafael José Iório, Jr., IMPA, Brazil, *rj.iorio@gmail.com*

Title: On the Cauchy Problem Associated to the Brinkman Equations: Some remarks

Abstract: We will present some comments and results pertaining the Cauchy Problem associated to the Brinkman Equations, that describe the flow of fluids in certain porous media, namely

$$\begin{cases} \phi \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = F(t, \rho), \quad x \in \Omega, \ t \geq 0 \\ (-\mu_{eff} \Delta + \frac{\mu}{k}) v = -\nabla P(\rho), \\ (\rho(0), v(0)) = (\rho_0, v_0), \end{cases} \tag{0.1}$$

where $\Omega \subseteq \mathbb{R}^n$, is an open set with a sufficiently smooth boundary, $\rho = \rho(t, x)$ is the the fluid density, $v = v(t, x)$ its velocity, P the pressure, and F is en external mass flow rate. The other symbols are physical constants that we will comment on briefly during the talk. We will consider local and global well-posedness of this system with several assumptions on Ω (which lead us to impose boundary conditions on the flow and in turn on the differential operators in question), and some non-trivial initial conditions.

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(12) Pedro Isaza, Universidad Nacional de Colombia, Medellin, Colombia, *pisaza@unal.edu.co*

Title: On the propagation of regularity of solutions of the Kadomtsev-Petviashvilli (KP-II) equation

Abstract: In this talk we consider the Cauchy problem for the Kadomtsev-Petviashvilli (KPII) equation

$$\begin{cases} \partial_t u + \partial_x^3 u + \alpha \partial_x^{-1} \partial_y^2 u + u\, \partial_x u = 0, & (x,y) \in \mathbb{R}^2,\ t > 0,\ \alpha = 1, \\ u(x,y,0) = u_0(x,y), \end{cases}$$

with initial datum u_0 such that $u_0,\ \partial_x^{-1} u_0 \in H^s(\mathbb{R}^2),\ s > 2$. We deduce some special regularity properties of the solutions to the problem. Mainly, we prove that if the restriction of u_0 to $(x_0, \infty) \times \mathbb{R}$ belongs to $H^m((x_0, \infty) \times \mathbb{R})$ for some $m \in \mathbb{Z}^+$, $m \geq 3$, and $x_0 \in \mathbb{R}$, then the restriction of the corresponding solution $u(\cdot, t)$ belongs to $H^m((\beta, \infty) \times \mathbb{R})$ for any $\beta \in \mathbb{R}$ and any $t > 0$.

This is a joint work with F. Linares and G. Ponce.

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(13) José Manuel Jiménez, Universidad Nacional de Colombia, Colombia, *jmjimene@unal.edu.co*

Title: The Initial Value Problem for the Zakharov-Kuznetsov equation in weighted sobolev spaces

Abstract: In this lecture we consider the initial value problem (IVP) associated to the two dimensional Zakharov-Kuznetsov equation

$$\begin{cases} u_t + \partial_x^3 u + \partial_x \partial_y^2 u + u \partial_x u = 0, & (x,y) \in \mathbb{R}^2,\ t \in \mathbb{R}, \\ u(x,y,0) = u_0(x,y). \end{cases}$$

We study the well-posedness of the IVP in the weighted Sobolev spaces

$$Z_{s,r} := H^s(\mathbb{R}^2) \cap L^2((1 + x^2 + y^2)^r dxdy), \quad s, r \in \mathbb{R}.$$

We prove that the IVP is locally well-posed in $Z_{s,s/2}$, for $s > 3/4$.

Joint work with Eddye Bustamante and Jorge Mejía.

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(14) Masaki Kurokiba, Muroran Institute of Technology, Japan, *kurokiba@mmm.muroran-it.ac.jp*

Title: Blowing up for a solution to system of the drift-diffusion equations in higher space dimensions

Abstract: We discuss the existence of the blow-up solution for multi-component parabolic-elliptic drift-diffusion model in higher space dimensions. We show that the local existence, uniqueness and well-posedness of a solution in the weighted L^2 spaces. Moreover we prove that if the initial data satisfies a certain condition, then the corresponding solution blows up in a finite time. This is a system case for the blow up result of the chemotactic and drift-diffusion equation proved by Nagai [8] and Nagai-Senba-Suzuki [9] and gravitational interaction of particles by Biler [2], Biler-Nadzieja [3], [4]. We generalize the result in Kurokiba-Ogawa [6], [7] and Kurokiba [5] for multi-component problem and give a sufficient condition for the finite time blow up of the solution.

This is joint work with Prof. Takayoshi Oagawa.

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(15) Felipe Linares, IMPA, Brazil, *linares@impa.br*

Title: On nonlinear dispersive equations

Abstract: In this talk we will discuss results regarding properties of solutions to some nonlinear dispersive equations such as well-posedness, decay, regularity and blow-up.

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(16) Luc Molinet, LMPT, Univ. Tours, France *Luc.Molinet@lmpt.univ-tours.fr*

Title: Improvement of the energy method for strongly non resonant dispersive equations and applications

Abstract: We propose a new approach to prove the local well-posedness of the Cauchy problem associated with strongly non resonant dispersive equations. As an example, we prove the well-posedness in the energy space of the Intermediate Long Wave and the Benjamin-Ono equations without using a gauge transform. It is worth noticing that our method of proof works on the torus as well as on the real line.

This is a joint work with Stéphane Vento (U. Paris 13).

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(17) Claudio Muñoz, Universidad de Chile, Chile, *cmunoz@dim.uchile.cl*

Title: Asymptotic stability for kinks of the ϕ^4 model.

Abstract: The purpose of this talk is to give some ideas of the proof of a recent result obtained in collaboration with M. Kowalczyk (U. Chile) and Y. Martel (E. Polytechnique), concerning the asymptotic stability, in the energy space, of kinks of a scalar field equation in 1+1 dimensions, usually referred as the ϕ^4 model. Without using normal forms methods, but only energy and virial identities, we show that perturbations of kinks must converge to zero locally and strongly in the energy space.

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(18) Fábio Natali, Universidade Estadual de Maringá, Brazil, *fmnatali@hotmail.com*

Title: The Fourth-order dispersive nonlinear Schrödinger equation: Orbital stability of a standing wave

Abstract: Considered in this talk is the one-dimensional forth-order dispersive cubic nonlinear Schrödinger equation with mixed dispersion. Orbital stability, in the energy space, of a particular standing-wave solution is proved in the context of Hamiltonian systems. The main result is established by constructing a suitable Lyapunov function.

This is a joint work with Ademir Pastor from IMECC/UNICAMP-Brazil.

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(19) Takaoshi Ogawa, Tohoku University, Japan, *ogawa@math.tohoku.ac.jp*

Title: Ill-posedness issue for quadratic nonlinear Schrödingier equations in lower space dimensions and related topics

Abstract: We consider the ill-posedness problem for quadratic nonlinear Schrödinger equation in lower space dimensions. The critical Sobolev scale for the quadratic polynomial nonlinearity is now identified and there is some inconsistency between the critical scaling exponent and space dimensions. In particular, two dimensional case seems to be exceptional. I am going to show the main reason why two dimensional case is exceptional and present some extended result to a system.

This talk is based on joint works with Tsukasa Iwabuchi and Kota Uriya.

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(20) Tohru Ozawa, Waseda University, Japan, *txozawa@waseda.jp*

Title: On the Hardy type inequalities

Abstract: We study the Hardy type inequalities in the framework of equalities. We present equalities which imply Hardy type inequalities by dropping remainders. A characterization of functions is given on vanishing remainders. This provides a simple and direct understanding of the Hardy type inequalities as well as the nonexistence of nontrivial extremizers.

This talk is based on a recent joint-work with Shuji Machihara and Hidemitsu Wadade.

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(21) Ademir Pazoto, Universidade Federal do Rio de Janeiro (UFRJ), Brazil, *ademir@im.ufrj.br*

Title: Stabilization of a dissipative Boussinesq system

Abstract: Our work considers a Boussinesq systems which couples two Benjamin-Bona-Mahony type equations posed on a bounded interval. We study the stabilization of the model when a localized damping term acts in one equation only. By means of spectral analysis and eigenvectors expansion of solutions, we prove that the energy associated to the model converges to zero as time goes to infinity. Also, we address the problem of unique continuation property for the corresponding conservative system.

Joint work with Sorin Micu from University of Craiova (Romania).

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(22) Didier Pilod, UFRJ, Brazil, *didierpilod@gmail.com*

Title: Asymptotic Stability of Zakharov-Kuznetsov solitons in dimension 2

Abstract: We prove that solitons of the Zakharov-Kuznetsov (ZK) equation, a physically relevant high dimensional generalization of the Korteweg-de Vries (KdV) equation appearing in Plasma Physics, and having mixed KdV and nonlinear Schrödinger (NLS) dynamics, are *strongly* asymptotically stable in the energy space. We also prove that the sum of well-arranged solitons is stable in the same space. Orbital stability of ZK solitons is well-known since the work of de Bouard in 1996. Our proofs extend the ideas by Martel and Merle for generalized KdV equations in one dimension to higher dimensions. In particular, we derive monotonicity properties for suitable *half-portions* of mass and energy; we also prove a new Liouville type property that characterizes ZK solitons, and a key Virial identity for the linear and nonlinear parts of the ZK dynamics. This last Virial identity relies on a simple sign condition, which is numerically verified in dimension 2.

This is a joint work with Raphaël Côte, Claudio Muñoz and Gideon Simpson.

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(23) Ramon Plaza, Universidad Nacional Autonoma, Mexico, *plaza@mym.iimas.unam.mx*

Title: On the spectral, modulational and orbital stability of perioidc wavetrains for the sine-Gordon equation

Abstract: In this talk I present a detailed analysis of stability properties of periodic traveling wave solutions of the nonlinear sine-Gordon equation. Stability is considered from the point of view of spectral analysis of the linearized problem (spectral stability analysis), from the point of view of wave modulation theory (the strongly nonlinear theory due to Whitham as well as the weakly nonlinear theory of wave packets), and in the orbital (nonlinear) framework. The connection between these results is made through a modulational instability index. We analyse waves of both subluminal and superluminal propagation velocities, as well as waves of both librational and rotational types. We prove that only subluminal rotational waves are spectrally stable and establish exponential instability in the other three cases. The proof corrects a frequently cited result given by Scott (1965). In addition, I will show that the spectral information is crucial in the establishment of the orbital stability for subluminal rotational sine-Gordon waves.

This is joint work with J. Angulo Pava, C.K.R.T. Jones, P.D. Miller and R. Marangell.

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(24) Gustavo Ponce, University of California-Santa Barbara, USA, *ponce@math.ucsb.edu*

Title: On special properties of solutions of the k-generalized Korteweg-de Vries equation

Abstract: In 1983 T. Kato established the so called "local smoothing effect" (LSE) in solutions of the KdV equation. Starting with the identity deduced by Kato to obtain the LSE in a joint work with P. Isaza and F. Linares we show that solutions of the IVP for the k-generalized KdV equation

$$\begin{cases} \partial_t u + \partial_x^3 u + u^k \partial_x u = 0, & t, \ x \in \mathbb{R}, \\ u(x,0) = u_0(x) \end{cases} \tag{0.2}$$

preserve some smoothness of the initial data u_0 and that this regularity moves with infinite speed to its left as time evolves:

Theorem 0.1. [1] *If* $u_0 \in H^{3/4^+}(\mathbb{R})$ *and for some* $l \in \mathbb{Z}^+, \ l \geq 1$ *and* $x_0 \in \mathbb{R}$

$$\| \partial_x^l u_0\|^2_{L^2((x_0,\infty))} = \int_{x_0}^{\infty} |\partial_x^l u_0(x)|^2 dx < \infty, \tag{0.3}$$

then the solution of the IVP (0.2) $u(x,t)$ *satisfies* : $\forall\, v > 0,\ \epsilon > 0$

$$\sup_{0\le t\le T} \int_{x_0+\epsilon-vt}^{\infty} (\partial_x^j u)^2(x,t)\,dx < c, \tag{0.4}$$

for $j = 0, 1, \ldots, l$ *with* $c = c(l; \|u_0\|_{3/4^+,2}; \|\,\partial_x^l u_0\|_{L^2((x_0,\infty))}; v; \epsilon; T)$.

Subsequent, in a joint work with F. Linares and D. Smith [4] it was proved that the solution flow of (0.2) does not preserve other kind of regularities exhibited by the initial data u_0. This is related with the "dispersive blow up" introduced by Bona-Saut [2] and the modification given by Linares-Scialom [3].

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(25) Svetlana Roudenko, George Washington University, USA, *roudenko@gwu.edu*

Title: Blow-up and scattering in the focusing NLKG and NLS equations

Abstract: We consider the focusing nonlinear Klein-Gordon and Schrodinger equations in the L^2-supercritical regime. One of the questions we investigate is the behavior of solutions above the energy (or mass-energy) threshold, i.e., when the energy of a solution exceeds the energy of the so-called ground state. We extend the known scattering versus blow-up dichotomy above the energy threshold for some set of finite variance solutions in the energy-subcritical and energy-critical regimes for the NLS equation, characterizing invariant sets of solutions (with either scattering or blow-up in finite time behavior) possibly with arbitrary large mass and energy. We then try to apply similar ideas to the Klein-Gordon equation.

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(26) Jorge Drumond Silva, Instituto Superior Técnico, Portugal, *jsilva@math.ist.utl.pt*

Title: On the global uniqueness for the Einstein-Maxwell-scalar field system with a cosmological constant

Abstract: In this talk, we present recent results on the future extendability of the maximal globally hyperbolic development for the Einstein-Maxwell-scalar field system with a cosmological constant, with spherically symmetric characteristic initial data given by a subextremal Reissner-Nordstrm black hole event horizon along the outgoing initial null hypersurface.

We will start by reviewing some basic notions of General Relativity, leading to the formulation of the celebrated strong cosmic censorship conjecture, from a PDE perspective. Then, we will describe the local and global well posedness of the characteristic problem above and study the stability of the radius function at the Cauchy horizon. Then, we show that, depending on the decay rate of the initial data, mass inflation may or may not occur. Under certain conditions, extensions of the spacetime can actually be obtained across the Cauchy horizon, as classical solutions of the Einstein equations.

We conclude by discussing how these results relate to the strong cosmic censorship conjecture.

This is joint work with João L. Costa, Pedro M. Girão and José Natário.

Abstracts - Posters

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(1) Giovana Alves, Universidade Estadual de Maringá, Brazil, *a_giovanaalves@yahoo.com.br*

Title: Orbital Stability of Periodic Waves for the Cubic-Quintic Schrödinger Type Equations

Abstract: In this work, we present results of existence, as well as, orbital stability of periodic standing waves for the cubic-quintic nonlinear Schrödinger type equations. The existence of periodic solutions and the spectral property related to the associated linearized operator will be done by using the arguments in [1]. Regarding the orbital stability, we adapt the recent development in [2] by constructing a suitable Lyapunov function.

Joint work with F. Natali.

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(2) Thiago Pinguello de Andrade, UTFPR, Brazil, *thiagopinguello@hotmail.com*

Title: Orbital Stability of periodic traveling waves for Gardner equation

Abstract: In this work we study the orbital stability of periodic traveling waves for the Gardner equation

$$v_t + v_{xxx} + avv_x + bv^2 v_x = 0, \tag{0.5}$$

where a and b are real parameters, with $b \neq 0$. Our strategy consists in applying a diffeomorphism that relates the solutions of the Gardner equation to that of the modified KdV,

$$u_t + u_{xxx} + \gamma 6u^2 u_x = 0, \tag{0.6}$$

with $\gamma = \text{sgn}(b)$. Such a diffeomophism is given by

$$(Tv)(x,t) := \sqrt{\frac{b}{6\gamma}}\left[v\left(x - \frac{a^2}{4b}t, t\right) + \frac{a}{2b}\right].$$

This allows us to transfer the problem of orbital stability for the Gardner equation to a equivalent problem for the modified KdV.

Joint work: Ademir Pastor Ferreira-IMECC-Unicamp

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(3) Isnaldo Isaac Barbosa, UFAL, Brazil, *isnaldo@pos.mat.ufal.br*

Title: About the Cauchy problem for nonlinear Schrödinger type interactions

Abstract: This study presents results of local and global well-posedenss for a system with quadratic coupling type Schrödinger equations. We will use the method-I and Bougain spaces to obtain the results presented here. The system studied here was treated in the periodic case by Angulo and Linares in [1].

This work is dedicated to the study of the Cauchy problem for a system modeling problems of nonlinear optics

$$\begin{cases} i\partial_t u(x,t) + p\partial_x^2 u(x,t) - \theta u(x,t) + \bar{u}(x,t)v(x,t) = 0, & x \in \mathbb{R},\ t \geq 0, \\ i\sigma\partial_t v(x,t) + q\partial_x^2 v(x,t) - \alpha v(x,t) + \frac{1}{2}u^2(x,t) = 0, & p,q = \pm 1, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \end{cases} \tag{0.7}$$

where u and v are functions that take on complex values, α and θ are real numbers representing physical parameters of the system, and $\sigma > 0$.

We observe that the model establishes the nonlinear coupling of two dispersive equations of Schrödinger type through quadratic terms

$$N_1(u,v) = \bar{u}v \quad \text{and} \quad N_2(u) = \frac{1}{2}u^2. \tag{0.8}$$

Physically, according to the work [1], the complex functions u and v represent packets of amplitudes of the first and second harmonic, of an optical wave, respectively. The values of p and q can be 1 or -1, depending on the signals provided by the relations between the dispersion/diffraction and the positive constant σ, which measures the magnitude ratios dispersion/diffraction. The interest in non-linear properties of optical materials has attracted the attention of physicists and mathematicians in recent years. Several studies suggest that exploring the nonlinear

response of matter, the ability bit-rate of optical fibers can be increased substantially; consequently, an improvement in the speed and economy of transmission and data manipulation. Particularly, in non-centrosymmetric materials (those that do not have symmetry inversion at the molecular level) the non-linear effects of lower order originate susceptibility of the second order, which means that the non-linear response for the electric field is quadratic order; see, for example, the article [3].

We started developing a local theory Bourgain spaces, following the techniques used in [2] and [4], where for each σ we get very positive overall results in Sobolev spaces with regularities not necessarily equal. Specifically, we will prove the results of well local place for data $(u_0, v_0) \in H^\kappa \times H^s$ with indexes $(\kappa, s) \in \mathcal{W}_\sigma$, where the flat region $\mathcal{W}_\sigma$ is defined as follows:

Definition 0.2. *Given $\sigma > 0$, we say that the pair of indices Sobolev (κ, s) verifies the hypothesis H_σ, if it meets one of the following conditions:*
a) $|\kappa| - 1/2 \leq s < \min\{\kappa + 1/2,\ 2\kappa + 1/2\}$ *para* $0 < \sigma < 2$;
b) $\kappa = s \geq 0$ *for* $\sigma = 2$;
c) $|\kappa| - 1 \leq s < \min\{\kappa + 1,\ 2\kappa + 1\}$ *for* $\sigma > 2$.

Using the previous definition set $\mathcal{W}_\sigma$ by

$$\mathcal{W}_\sigma = \Big\{(\kappa, s) \in \mathbb{R}^2;\ (\kappa, s) \text{ verifies the hypothesis } H_\sigma\Big\}. \tag{0.9}$$

We ended using the method-I to globally extend local solutions obtained for data $H^s \times H^s$ with $s \leq 0$. Specifically, $-1/4 \leq s \leq 0$ for $0 < \sigma < 2$ and $-1/2 \leq s \leq 0$ for $\sigma > 2$. At this point, it will be crucial to use Strichartz estimates, refined in Bourgain spaces for the Schrödinger equation.

In short, we get the following local results:

Theorem 0.3. *Given $\sigma > 0$ and $(u_0, v_0) \in H^\kappa \times H^s$ with $(\kappa, s) \in \mathcal{W}_\sigma$, defined by (0.9). The problem of Cauchy (0.7) is locally well-posedness $H^\kappa \times H^s$ in the following sense: for each $\rho > 0$, there $T = T(\rho) > 0$ and $b > 1/2$ such that for all initial data with $\|u_0\|_{H^\kappa} + \|v_0\|_{H^s} < \rho$, there is only one solution (u, v) for (0.7) satisfying the following conditions:*

$$\psi_T(t)u \in X^{\kappa,b} \quad e \quad \psi_T(t)v \in X^{s,b}_\sigma, \tag{0.10}$$

$$u \in C\big([0,T]; H^\kappa\big) \quad e \quad v \in C\big([0,T]; H^s\big). \tag{0.11}$$

Moreover, the application data-solution is locally Lipschitz.

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This work is part of my doctoral thesis under supervision of Dr. Adán Corcho.

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(4) Márcio Cavalcante de Melo, UFRJ, Brazil, *marciocavalcante1988@gmail.com*

Title: The initial-boundary value problem for Schrödinger-KdV system on the half line

Abstract: We prove, by adapting the method of Colliander-Kenig [1], local well-posedness of the initial-boundary value problem for the Schrödinger-KdV system on the half-line under low boundary regularity assumptions. We also use the ideas contained in [2], [3] and [4]. The main new ingredient is the introduction of a analytic family of boundary forcing operators extending the single operator introduced in [3] and the study of these operators in Bourgain spaces.

Remark 0.4. *The content of this work is part of the author's Ph.D. Thesis at the Universidade Federal do Rio de Janeiro under direction of Professor Adán J. Corcho. The author is partially supported by CAPES.*

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(5) Luis Andrés Rosso Cerón, USP, Brazil *andresrosso323@hotmail.com*

Title: Nonlinear Schrödinger equation with two defect symmetrical points

Abstract: We study analytically the existence and orbital stability of the peak-standing-wave solutions for the cubic nonlinear Schrödinger equation with two interactions points. This equation admits a least two smooth curve of positive solutions with a profile given by the Jacobi elliptic function of dnoidal and cnoidal type between the defects point. Via a perturbation method and continuation argument, we obtain that in the case of an attractive defect (dnoidal case) the standing wave solutions are stable in H^1_{per}.

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(6) Fabrício Cristófani, UEM, PR, Brasil, *fabriciognr@gmail.com*

Title: Orbital Stability of Periodic Traveling Wave Solutions of the Generalized Kawahara Equation

Abstract: In this talk, we investigate the orbital stability of periodic traveling waves for the Kawahara type equation as

$$u_t + u^p u_x + \lambda u_{xxx} - u_{xxxxx} = 0,$$

where $u = u(x,t)$ is a real valued function with domain $\mathbb{R}\times\mathbb{R}$, $p = 1, 2$ and $\lambda = 0, 1$.

We prove that the periodic traveling wave, under certain conditions, minimizes a convenient functional by using an adaptation of the method developed by Grillakis, Shatah e Strauss (See [2]). The required spectral properties for the orbital stability were obtained by knowing the positiveness of Fourier transform associated with the periodic wave combined with the approach [1].

This is a joint work with Fábio Natali from DMA/UEM.

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(7) Leandro Domingues, UFES , Brazil, *leandro.ceunes@gmail.com*

Title: Sharp Well-Posedness Results for a Schrödinger-Benjamin-Ono System

Abstract: In this work we study the Cauchy problem for the coupled Schrödinger-Benjamin-Ono system

$$\begin{cases} i\partial_t u + \partial_x^2 u = \alpha u v, & t \in [-T, T],\ x \in \mathbb{R}, \\ \partial_t v + \nu \mathcal{H} \partial_x^2 v = \beta \partial_x (|u|^2), & \\ u(0,x) = \phi,\ v(0,x) = \psi, & (\phi, \psi) \in H^s(\mathbb{R}) \times H^{s'}(\mathbb{R}). \end{cases}$$

In the *non-resonant* case ($|\nu| \neq 1$), we prove local well-posedness for a large class of initial data. This improves the results obtained by Bekiranov, Ogawa and Ponce (1998). Moreover, we prove C^2-*ill-posedness* at *low-regularity*, and also when the difference of regularity between the initial data is large enough (i.e. $|s - s'| > c$). As far as we know, this last ill-posedness result is the first of this kind for a non-linear dispersive system. Finally, we also prove that the local well-posedness result obtained by Pecher (2006) in the *resonant* case ($|\nu| = 1$) is sharp except for the end-point.

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(8) Fernando Galego, UFRJ, Brazil, *ferangares@gmail.com*

Title: Global well-posedness and exponential stability for the Generalized Kdv-Burgers Equation

Abstract: In this paper we investigated the well-posedness and the exponential stability of the generalized Korteweg-de Vries Burgers (GKdv-B) equation

$$u_t - u_{xx} + u_{xxx} + a(u)u_x + b(x)u = 0, \tag{0.12}$$

on the whole real line under the effect of a damping term. We obtain the global well-posedness in $H^s(\mathbb{R})$ for $1 \leq p < 2$ and $s \in [0, 3]$ and in $L^2(\mathbb{R})$ for $2 \leq p < 5$ and $H^3(\mathbb{R})$ for $p \geq 2$. Here, p denotes the exponent in the nonlinear term. The exponential stabilization is obtained for a definite damping term ($1 \leq p < 2$) by using multiplier techniques combined with interpolation. Under the effect of a localized damping term ($2 \leq p < 5$) we obtain a similar result by multiplier techniques combined with compactness arguments, reducing the problem to prove a unique

continuation property for weak solutions.

Joint work with A. F. Pazoto, Universidade Federal do Rio de Janeiro.

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(9) Carlos M. Guzman, Instituto de Matemática , UFMG, Brazil, *charlis.100@gmail.com*

Title: Scattering for inhomogeneous nonlinear Schrdinger equations

Abstract: The purpose of this work is to study scattering for the inhomogeneous nonlinear Schrödinger equation (INLS) in $H^1(\mathbb{R}^N)$. To do this, we use the ideas introduced by Kenig-Merle [5] in their study of the energy-critical NLS and Holmer-Roudenko [4] for the $3D$ cubic NLS.

We consider the initial value problem (IVP) or also called the Cauchy problem for the inhomogenous nonlinear Schrödinger equation (INLS)

$$\begin{cases} i\partial_t u + \Delta u + \lambda |x|^{-b}|u|^{\alpha}u = 0, & t \in \mathbb{R},\ x \in \mathbb{R}^N, \\ u(0,x) = u_0(x), \end{cases} \tag{0.13}$$

where $u = u(t,x)$ is a complex-valued function in space-time $\mathbb{R} \times \mathbb{R}^N$ and $\alpha, b > 0$. The equation is called focusing INLS for $\lambda = +1$ and defocusing INLS for $\lambda = -1$.

The scale-invariant Sobolev norm is $H^{s_c}(\mathbb{R}^N)$ with $s_c = \frac{N}{2} - \frac{2-b}{\alpha}$ (Critical Sobolev index). If $s_c = 0$ (alternatively $\alpha = \frac{4-2b}{N}$) the problem is known as the mass-critical or L^2-critical; when $s_c = 1$ (alternatively $\alpha = \frac{4-2b}{N-2}$) it is called energy-critical or $\dot{H}^1$-critical, finally the problem is known as mass-supercritical and energy-subcritical if $0 < s_c < 1$. The Cauchy problem for the INLS (0.13) was already studied for many authors in recent years. Let us briefly recall the best results available in the literature. Cazenave studied the well-posedness using the abstract theory. To do this, he analized (0.13) in the sense of distributions, that is, $i\partial_t u + \Delta u + |x|^{-b}|u|^{\alpha}u = 0$ in $H^{-1}(\mathbb{R}^N)$ for a.a. $t \in I$, and using some results of Functional Analysis and Semigroups of Linear Operators, he proved that it is appropriate to seek solutions of (0.13)

$$u \in C\left([0,T), H^1(\mathbb{R}^N)\right) \cap C^1\left([0,T), H^{-1}(\mathbb{R}^N)\right) \text{ for some T} > 0,$$

for the defocusing case ($\lambda = -1$) any local solution of the (IVP) (0.13) with $u_0 \in H^1(\mathbb{R}^N)$ extends globally in time.

Other authors like Genoud and Stuart also studied this problem for the focusing case ($\lambda = 1$). They showed the IVP (0.13) is well-posed in $H^1(\mathbb{R}^N)$

- locally if $0 < \alpha < 2^* := \begin{cases} \frac{4-2b}{N-2} & N \geq 3; \\ \infty & N = 1, 2; \end{cases}$
- globally for small initial conditions if $\frac{4-2b}{N} < \alpha < \frac{4-2b}{N-2}$;
- globally for any initial condition in $H^1(\mathbb{R}^{\mathbb{N}})$ if $0 < \alpha < \frac{4-2b}{N}$.

Another interesting problem to (0.13) is scattering. For the $3D$ defocusing NLS ((0.13) with $b = 0$), scattering has been established for all H^1 solutions (regardless of size) by Ginibre-Velo using a Morawetz inequality. This proof was simplified by Colliander-Keel-Staffilani-Takaoka-Tao using a new interaction Morawetz inequality they discovered. Other authors like Killip-Tao-Visan, Tao-Visan-Zhang and Killip-Visan-Zhang extend for the L^2-critical defocusing NLS in arbitrary dimension $N \geq 1$. These Morawetz estimates, however, are not positive definite for solutions to the focusing case and cannot be used to study the scattering problem in this case.

In 2006, Kenig and Merle [5] developed a new method to show scattering for the focusing NLS. They proved scattering in $H^1(\mathbb{R}^N)$ in this case with initial data in $\dot{H}^1(\mathbb{R}^N)$ in dimensions $N = 3, 4, 5$. To do this, they applied the concentration-compactness and rigidity technique. The concentration-compactness method appears in the context of wave equation in Gerard and NLS in Keraani. The rigidity argument (estimates on a localized variance) is a technique developed by F. Merle in mid 1980's.

For the L^2-supercritical and H^1-subcritical case several scattering results were obtained for the focusing NLS in $H^1(\mathbb{R}^N)$. Holmer-Roudenko [4] showed scattering to $3D$ NLS for radial initial data, then Duyckaerts-Holmer-Roudenko [1] extended this result for the nonradial data. Recently, Fang-Xie-Cazenave and Cristi Guevara [3] generalized the above result for arbitrary $N \geq 1$.

In the spirit of Holmer, Roudenko, Duyckaerts and Guevara, we show scattering in $H^1(\mathbb{R}^N)$ for the focusing inhomogeneous nonlinear Schrödinger equation (0.13) in the case $0 < s_c < 1$ (mass-supercritical and energy subcritical equation). equation). Our aim in this work is to show the following result:

Theorem 0.5. *Let* $u_0 \in H^1(\mathbb{R}^N)$*, and let* u *the corresponding solution to* (0.13) *in* H^1*. Suppose*

$$M[u]^{s_c} E[u]^{1-s_c} < M[Q]^{s_c} E[Q]^{1-s_c},$$

and if $\|u_0\|^{s_c}\|\nabla u_0\|^{1-s_c} < \|Q\|^{s_c}\|\nabla Q\|^{1-s_c}$*, then* u *scatters in* H^1*. That is, there exists* $v \in H^1(\mathbb{R}^N)$ *such that*

$$\lim_{t\to\infty} \|u(t) - U(t)v\|_{H^1} = 0,$$

where Q *is the ground state of* $-Q + \Delta Q + \lambda|x|^{-b}|Q|^{\alpha}Q = 0$.

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