~~OITAVA~~ LISTA DE EXERCÍCIOS DE CÁLCULO 1 – CURSÃO

1. Differentiate each of the following functions (remember that a^{b^c} always denotes $a^{(b^c)}$).

(i) $f(x) = e^{e^{e^{e^x}}}$.
(ii) $f(x) = \log(1 + \log(1 + \log(1 + e^{1+e^{1+x}})))$.
(iii) $f(x) = (\sin x)^{\sin(\sin x)}$.
(iv) $f(x) = e^{\left(\int_0^x e^{-t^2}\,dt\right)}$.
(v) $f(x) = \sin x^{\sin x^{\sin x}}$.
(vi) $f(x) = \log_{(e^x)} \sin x$.
(vii) $f(x) = \left[\arcsin\left(\frac{x}{\sin x}\right)\right]^{\log(\sin e^x)}$.
(viii) $f(x) = (\log(3 + e^4))e^{4x} + (\arcsin x)^{\log 3}$.
(ix) $f(x) = (\log x)^{\log x}$.
(x) $f(x) = x^x$.

4. Graph each of the following functions.

(a) $f(x) = e^{x+1}$.
(b) $f(x) = e^{\sin x}$.
(c) $f(x) = e^x + e^{-x}$.
(d) $f(x) = e^x - e^{-x}$.
(Compare the graph with the graphs of exp and $1/\exp$.)
(e) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^{2x} - 1}{e^{2x} + 1} = 1 - \dfrac{2}{e^{2x} + 1}$.

10. Show that

$$F(x) = \int_2^x \frac{1}{\log t}\,dt$$

is not bounded on $[2, \infty)$.

11. Let f be a nondecreasing function on $[1, \infty)$, and define

$$F(x) = \int_1^x \frac{f(t)}{t}\,dt, \qquad x \geq 1.$$

Prove that f is bounded on $[1, \infty)$ if and only if $F/\log$ is bounded on $[1, \infty)$.

12. Find

(a) $\lim_{x\to\infty} a^x$ for $0 < a < 1$. (Remember the definition!)

(b) $\lim_{x\to\infty} \dfrac{x}{(\log x)^n}$.

(c) $\lim_{x\to\infty} \dfrac{(\log x)^n}{x}$.

(d) $\lim_{x\to 0^+} x(\log x)^n$. Hint: $x(\log x)^n = \dfrac{(-1)^n \left(\log \dfrac{1}{x}\right)^n}{\dfrac{1}{x}}$.

(e) $\lim_{x\to 0^+} x^x$.

14. (a) Find the minimum value of $f(x) = e^x/x^n$ for $x > 0$, and conclude that $f(x) > e^n/n^n$ for $x > n$.

(b) Using the expression $f'(x) = e^x(x - n)/x^{n+1}$, prove that $f'(x) > e^{n+1}/(n+1)^{n+1}$ for $x > n + 1$, and thus obtain another proof that $\lim_{x\to\infty} f(x) = \infty$.

***23.** Prove that if $f(x) = \displaystyle\int_0^x f(t)\,dt$, then $f = 0$.

28. (a) Let f and g be continuous nonnegative functions on $[a, b]$, and let $C > 0$. Suppose that

$$f(x) \leq C + \int_a^x fg \qquad a \leq x \leq b.$$

Prove *Gronwall's inequality*:

$$f(x) \leq Ce^{\int_a^x g}.$$

Hint: Consider the derivative of the function $h(x) = C + \int_a^x fg$.