are
ues
(35)
^{2} is
ions
(36)
(37)
), we
(38)
(39)
oblem
(40)
(41)

Finally, we point out that while w
(12a)–(12c) and (32a)–(32c) for a rod of
ture $u(x, t)$ within the infinite slab $0 \leqq$
initial temperature $f(x)$ depends only up
are either both insulated or both held a

8.5 PROBLEMS

Solve the boundary value problems in Problems 1–12.

1. $u_t = 3u_{xx}$, $0 < x < \pi$, $t > 0$; $u(0, t) = u(\pi, t) = 0$, $u(x, 0) = 4 \sin 2x$

2. $u_t = 10u_{xx}$, $0 < x < 5$, $t > 0$; $u_x(0, t) = u_x(5, t) = 0$, $u(x, 0) = 7$

3. $u_t = 2u_{xx}$, $0 < x < 1$, $t > 0$; $u(0, t) = u(1, t) = 0$, $u(x, 0) = 5 \sin \pi x - \frac{1}{5} \sin 3\pi x$

4. $u_t = u_{xx}$, $0 < x < \pi$, $t > 0$; $u(0, t) = u(\pi, t) = 0$, $u(x, 0) = 4 \sin 4x \cos 2x$

5. $u_t = 2u_{xx}$, $0 < x < 3$, $t > 0$; $u_x(0, t) = u_x(3, t) = 0$, $u(x, 0) = 4 \cos \frac{2}{3}\pi x - 2 \cos \frac{4}{3}\pi x$

6. $2u_t = u_{xx}$, $0 < x < 1$, $t > 0$; $u(0, t) = u(1, t) = 0$, $u(x, 0) = 4 \sin \pi x \cos^3 \pi x$

7. $3u_t = u_{xx}$, $0 < x < 2$, $t > 0$; $u_x(0, t) = u_x(2, t) = 0$, $u(x, 0) = \cos^2 2\pi x$

8. $u_t = u_{xx}$, $0 < x < 2$, $t > 0$; $u_x(0, t) = u_x(2, t) = 0$, $u(x, 0) = 10 \cos \pi x \cos 3\pi x$

9. $10u_t = u_{xx}$, $0 < x < 5$, $t > 0$; $u(0, t) = u(5, t) = 0$, $u(x, 0) = 25$

10. $5u_t = u_{xx}$, $0 < x < 10$, $t > 0$; $u(0, t) = u(10, t) = 0$, $u(x, 0) = 4x$

11. $5u_t = u_{xx}$, $0 < x < 10$, $t > 0$; $u_x(0, t) = u_x(10, t) = 0$, $u(x, 0) = 4x$

12. $u_t = u_{xx}$, $0 < x < 100$, $t > 0$; $u(0, t) = u(100, t) = 0$, $u(x, 0) = x(100 - x)$

13. Suppose that a rod 40 cm long with insulated lateral surface is heated to a uniform temperature of 100°C, and that at time $t = 0$ its two ends are embedded in ice at 0°C. (a) Find the formal series solution for the temperature $u(x, t)$ of the rod. (b) In the case the rod is made of copper, show that after 5 min the temperature at its midpoint is about 15°C. (c) In the case the rod is made of concrete, use the first term of the series to find th
15°C.

14. A copper ro
has initial tempe
ends are insulate
ture be at x = 1
long will its tem

15. The two face
ture zero, and th
$u(x, 0) = A$ (a co
$x < L$. Derive th

$u(x, t) =$

16. Two iron sla
temperature 100
ture 0°C. At time
outer faces are k
to find that after
face is approxim
are instead made
common face re

17. (Steady-stat
insulated rod wi
fixed endpoint te
It is observed em
a **steady-state te**
$u_t = 0$ in the bo
solution of the e

$\frac{\partial^2 u_{ss}}{\partial x^2} =$

Find $u_{ss}(x)$. (b) T
to be

If, finally, a string has both an in
an initial velocity $y_t(x, 0) = g(x)$, w
$y(x, t)$ by adding the d'Alembert so
Eqs. (30) and (37), respectively. Hence
initial conditions are described by

$$y(x, t) = \frac{1}{2}[F(x + at) + F(x - at$$

a superposition of four waves moving
ing to the left and two to the right.

8.6 PROBLEMS

Solve the boundary value problems in Problems 1–10.

1. $y_{tt} = 4y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0, t) = y(\pi, t) = 0$, $y(x, 0) = \frac{1}{10}\sin 2x$, $y_t(x, 0) = 0$

2. $y_{tt} = y_{xx}$, $0 < x < 1$, $t > 0$; $y(0, t) = y(1, t) = 0$, $y(x, 0) = \frac{1}{10}\sin \pi x - \frac{1}{20}\sin 3\pi x$, $y_t(x, 0) = 0$

3. $4y_{tt} = y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0, t) = y(\pi, t) = 0$, $y(x, 0) = y_t(x, 0) = \frac{1}{10}\sin x$

4. $4y_{tt} = y_{xx}$, $0 < x < 2$, $t > 0$; $y(0, t) = y(2, t) = 0$, $y(x, 0) = \frac{1}{5}\sin \pi x \cos \pi x$, $y_t(x, 0) = 0$

5. $y_{tt} = 25y_{xx}$, $0 < x < 3$, $t > 0$; $y(0, t) = y(3, t) = 0$, $y(x, 0) = \frac{1}{4}\sin \pi x$, $y_t(x, 0) = 10\sin 2\pi x$

6. $y_{tt} = 100y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0, t) = y(\pi, t) = 0$, $y(x, 0) = x(\pi - x)$, $y_t(x, 0) = 0$

7. $y_{tt} = 100y_{xx}$, $0 < x < 1$, $t > 0$; $y(0, t) = y(1, t) = 0$, $y(x, 0) = 0$, $y_t(x, 0) = x$

8. $y_{tt} = 4y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0, t) = y(\pi, t) = 0$, $y(x, 0) = \sin x$, $y_t(x, 0) = 1$

9. $y_{tt} = 4y_{xx}$, $0 < x < 1$, $t > 0$; $y(0, t) = y(1, t) = 0$, $y(x, 0) = 0$, $y_t(x, 0) = x(1 - x)$

10. $y_{tt} = 25y_{xx}$, $0 < x < \pi$, $t > 0$; $y(0, t) = y(\pi, t) = 0$, $y(x, 0) = y_t(x, 0) = \sin^2 x$

11. Suppose that a string 2 ft long weighs $\frac{1}{32}$ oz and is subjected to a tension of 32 lb. Find the fundamental frequency with which it vibrates and the velocity with which the vibration waves travel along it.

12. Show that the amplitude of the oscillations of the midpoint of the string of Example 3 is

$$y\left(\frac{L}{2}, \frac{L}{2a}\right) = \frac{4v_0L}{\pi^2 a}\sum_{n \text{ odd}}\frac{1}{n^2} = \frac{v_0L}{2a}.$$

If the string is
of the pickup
approximately

13. Suppose th
all x. Use the c
$F(x + at)$ and
a^2y_{xx}.

14. Given the
show that the
satisfies the co
and $y_t(x, 0) =$

15. If $y(x, 0) =$

$$y(x, t) =$$

Make appropr
Eqs. (37) and

16. (a) Show
transform the
(b) Conclude t
$y(x, t) = F(x$
waves travelin

17. Suppose t
Square the
termwise—ap
functions—to

$$E = \frac{1}{2}$$

18. Consider
$x = 0$ is fixed,