Questão 1

$$y'' + y = u_{\pi/2}(t) + 3\delta\left(t - \frac{3\pi}{2}\right) \qquad y(0) = y'(0) = 0$$

$$s^2 Y(s) - s\,y(0) - y'(0) + Y(s) = \frac{e^{-\frac{\pi}{2}s}}{s} + 3e^{-\frac{3\pi}{2}s}$$

$y(0) \to 0$, $y'(0) \to 0$

$$\boxed{Y(s) = \frac{e^{-\frac{\pi}{2}s}}{s(s^2+1)} + \frac{3e^{-\frac{3\pi}{2}s}}{s^2+1}}$$ (0,8)

$$\mathcal{L}^{-1}\left\{\frac{\frac{1}{s^2+1}}{s}\right\} = \int_0^t \operatorname{sen}\tau \, d\tau = -\cos t + 1$$ (0,3)

$$\mathcal{L}^{-1}\left\{e^{-\frac{\pi}{2}s}\frac{1}{s(s^2+1)}\right\} = u_{\pi/2}(t)\left[-\cos\left(t - \frac{\pi}{2}\right) + 1\right]$$ (0,3)

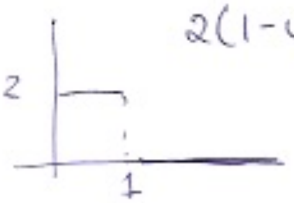
$$3\mathcal{L}^{-1}\left\{e^{-\frac{3\pi}{2}s}\frac{1}{s^2+1}\right\} = 3u_{\frac{3\pi}{2}}(t)\left[\operatorname{sen}\left(t - \frac{3\pi}{2}\right)\right]$$ (0,3)

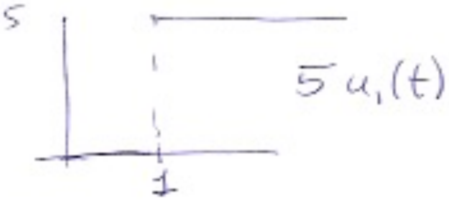
$$\boxed{y(t) = -u_{\pi/2}(t)\cos\left(t - \frac{\pi}{2}\right) + u_{\pi/2}(t) + 3u_{\frac{3\pi}{2}}(t)\operatorname{sen}\left(t - \frac{3\pi}{2}\right)}$$

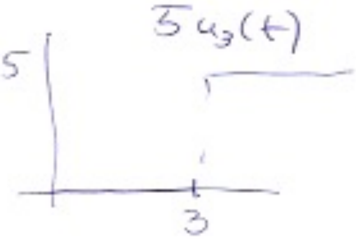
Questão 2

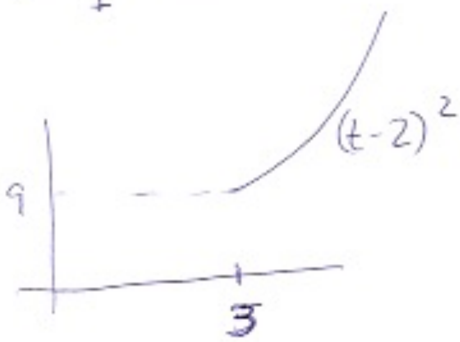
$$f(t) = \begin{cases} 2 & 0 \le t < 1 \\ 5 & 1 \le t < 3 \\ (t-2)^2 & 3 \le t < 5 \\ 0 & t \geq 5 \end{cases} \quad \underline{\text{ou}} \quad f(x) = \begin{cases} 2 & 0 \le t < 1 \\ 5 & 1 \le t < 3 \\ (t-2)^2 & 3 \le t \end{cases}$$

ambas extensões foram aceitas









$u_3(t)(t-2)^2$ ou

adicionar $-u_5(t)(t-2)^2$

a) $f(t) = 2(1-u_1(t)) + 5(u_1(t)-u_3(t)) + u_3(t)(t-2)^2$ (0.3) (0.3) (0.4)

b) $\mathcal{L}\{f(t)\} = \frac{2}{s} + \frac{3e^{-s}}{s} - \frac{5e^{-3s}}{s} + e^{-3s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)$ (0.6) (0.4)

$\mathcal{L}\{u_3(t)\, g(t-3)\} = e^{-3s}G(s) = e^{-3s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right)$

$g(t-3) = (t-2)^2$

$v = t-3 \Rightarrow t = v+3$

$g(v) = (v+1)^2 = v^2 + 2v + 1$

ou p/ a) $2(1-u_1(t)) + 5(u_1(t)-u_3(t)) + (t-2)^2(u_3-u_5)$ [0.3] [0.3] [0.4]

$\mathcal{L}\{f(t)\} = \frac{2}{s} + \frac{3e^{-s}}{s} - \frac{5e^{-3s}}{s} + e^{-3s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) + \left[-e^{-5s}\left(\frac{2}{s^3} + \frac{12}{s^2} + \frac{9}{s}\right)\right]$ [0.4] [0.6]

$\mathcal{L}\{u_5(t)\, h(t-5)\} = e^{-5s}H(s)$

$h(t-5) = (t-2)^2$

$v = t-5 \quad t = v+5$

$h(v) = (v+3)^2 = v^2 + 6v + 9$

$X' = AX$

(3) $|A - \lambda I| = 0$

a) $\begin{vmatrix} -2-\lambda & \alpha \\ 1 & -4-\lambda \end{vmatrix} = 0$

$(-2-\lambda)(-4-\lambda) - \alpha = 0.$ (05)

$8 + \lambda^2 + 6\lambda - \alpha = 0 = \lambda^2 + 6\lambda + (8-\alpha)$

$\dfrac{-6 \pm \sqrt{36 - 4(8-\alpha)}}{2} = -3 \pm \dfrac{\sqrt{4+4\alpha}}{2}$

$\Delta = 4 + 4\alpha = 0$ $\boxed{\alpha = -1}$ (05)

$\boxed{\lambda = -3}$

b) $\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$

$v_1 - v_2 = 0$ $v_1 = 1$ $v_2 = 1$

$\boxed{X_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}}$ (03)

$(A - \lambda I) W = V$

$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ (04)

$w_1 - w_2 = 1$ $w_1 = 2$ $w_2 = 1$

$\boxed{X_2 = \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} t + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right] e^{-3t}}$ (03)

$X(t) = c_1 X_1(t) + c_2 X_2(t).$

Gabarito

4- Matriz fundamental:

$$\phi = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix} \qquad (0,3)$$

$$\det \phi = e^{-t} + 4e^{-t} = 5e^{-t}$$

$$\phi^{-1} = \frac{1}{5e^{-t}} \begin{pmatrix} e^{2t} & -e^{2t} \\ 4e^{-3t} & e^{-3t} \end{pmatrix} \qquad (0,3)$$

$$U' = \phi^{-1} F = \frac{1}{5} \begin{pmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{pmatrix} \begin{pmatrix} e^{3t} + 1 \\ e^{t} \end{pmatrix}$$

$$= \frac{1}{5} \begin{pmatrix} e^{6t} + e^{3t} - e^{4t} \\ 4e^{t} + 4e^{-2t} + e^{-t} \end{pmatrix} \qquad (0,5)$$

$$\Rightarrow U(t) = \frac{1}{5} \begin{pmatrix} \frac{e^{6t}}{6} + \frac{e^{3t}}{3} - \frac{e^{4t}}{4} \\ 4e^{t} + \frac{4e^{-2t}}{-2} - e^{-t} \end{pmatrix} \qquad (0,5)$$

$$\Rightarrow X(t) = C_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \phi(t) \cdot U(t)$$

(0,4)

Questão 5. (a) Seja $f(x) = \frac{\ln x}{\sqrt{x}}$, $x > 0$. Então,

$$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = \lim_{x\to\infty} f(x). \qquad (0,2)$$

Como $\lim_{x\to\infty} \ln x = \infty$ e $\lim_{x\to\infty} \sqrt{x} = \infty$, podemos aplicar a regra de L'Hospital (0,2), para concluir que

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to\infty} \frac{2\sqrt{x}}{x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}} = 0.$$

Logo,

$$\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = 0. \qquad (0,2)$$

(b) Observe que se $\{s_n\}$ é a sequência das somas parciais então

$$s_n = \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \ldots + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right) + \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}. \qquad (0,4)$$

Assim,

$$\lim_{n\to\infty} s_n = \frac{5}{6},$$

e a soma da série é, portanto, $s = 5/6$. (0,3)

(c) Vamos aplicar o Teste da Razão. Seja $a_k = (-1)^k \frac{(k!)^3}{(3k)!}$. Temos

$$\left|\frac{a_{k+1}}{a_k}\right| = \frac{((k+1)!)^3}{(3(k+1))!} . \frac{(3k)!}{(k!)^3} = \frac{(k+1)^3 (k!)^3 (3k)!}{(3k+3)(3k+2)(3k+1)(3k)!(k!)^3}$$

$$= \frac{(k+1)^3}{(3k+3)(3k+2)(3k+1)}. \qquad (0,3)$$

Portanto, dividindo numerador e denominador por k^3,

$$\lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty} \frac{\left(1+\frac{1}{k}\right)^3}{\left(3+\frac{3}{k}\right)\left(3+\frac{2}{k}\right)\left(3+\frac{1}{k}\right)} = \frac{1}{27} < 1. \qquad (0,2)$$

Pelo Teste da Razão a série é absolutamente convergente.(0,2)