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*Erratum*

# Moduli Spaces of Self-Dual Connections over Asymptotically Locally Flat Gravitational Instantons

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As it was pointed out by U. Bunke, there is an error in the formulation and proof of Lemma 2.1 in our paper [2]. Hereby we would like to correct it. For sake of clarity we present the whole correctly formulated lemma and its proof.

**Lemma 2.1.** *Fix an $0 < \rho < \varepsilon$ and let $\nabla_{A_\rho} = \mathrm{d} + A_\rho$ and $\nabla_{B_\rho} = \mathrm{d} + B_\rho$ be two smooth $SU(2)$ connections in a fixed smooth gauge on the trivial $SU(2)$ bundle $E|_{\partial M_\rho}$. Then there is a constant $c_1 = c_1(B_\rho) > 0$, depending on $\rho$ only through $B_\rho$, such that*

$$|\tau_{\partial M_\rho}(A_\rho) - \tau_{\partial M_\rho}(B_\rho)| \leq c_1 \|A_\rho - B_\rho\|_{L^2_{1,B_\rho}(\partial M_\rho)}$$

*that is, the Chern–Simons functional is continuous in the $L^2_{1,B_\rho}$ norm.*

*Moreover, for each $\rho$, $\tau_{\partial M_\rho}(A_\rho)$ is constant on the path connected components of the character variety $\chi(\partial M_\rho)$.*

*Proof.* The first observation follows from the identity

$$\tau_{\partial M_\rho}(A_\rho) - \tau_{\partial M_\rho}(B_\rho)$$
$$= -\frac{1}{8\pi^2} \int\limits_{\partial M_\rho} \mathrm{tr}\left( (F_{A_\rho} + F_{B_\rho}) \wedge (A_\rho - B_\rho) - \frac{1}{3}(A_\rho - B_\rho) \wedge (A_\rho - B_\rho) \wedge (A_\rho - B_\rho) \right),$$

which implies that there is a constant $c_0 = c_0(\rho, B_\rho)$ such that

$$|\tau_{\partial M_\rho}(A_\rho) - \tau_{\partial M_\rho}(B_\rho)| \leq c_0 \|A_\rho - B_\rho\|_{L^{\frac{3}{2}}_{1,B_\rho}(\partial M_\rho)}$$

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that is, the Chern–Simons functional is continuous in the $L^{\frac{3}{2}}_{1,B_\rho}$ norm. A standard application of Hölder's inequality on $(\partial M_\rho, \tilde{g}|_{\partial M_\rho})$ then yields

$$\|A_\rho - B_\rho\|_{L^{\frac{3}{2}}_{1,B_\rho}(\partial M_\rho)} \leq \sqrt{2}\left(\mathrm{Vol}_{\tilde{g}|_{\partial M_\rho}}(\partial M_\rho)\right)^{\frac{1}{6}} \|A_\rho - B_\rho\|_{L^2_{1,B_\rho}(\partial M_\rho)}.$$

The metric locally looks like $\tilde{g}|_{\partial M_\rho \cap U^*_\varepsilon} = \rho^2\varphi(\mathrm{d}u^2+\mathrm{d}v^2)+\rho^4(\mathrm{d}\tau^2+2h_{\tau,u}\mathrm{d}\tau\mathrm{d}u+\dots)$ with $\varphi$ and $h_{\tau,u}$, etc. being bounded functions of $(u, v, \rho)$ and $(u, v, \rho, \tau)$ respectively, hence the metric coefficients as well as the volume of $(\partial M_\rho, \tilde{g}|_{\partial M_\rho})$ are bounded functions of $\rho$, consequently we can suppose that $c_1$ does not depend explicitly on $\rho$.

Concerning the second part, assume $\nabla_{A_\rho}$ and $\nabla_{B_\rho}$ are two smooth, flat connections belonging to the same path connected component of $\chi(\partial M_\rho)$. Then there is a continuous path $\nabla_{A^t_\rho}$ with $t \in [0, 1]$ of flat connections connecting the given flat connections. Out of this we construct a connection $\nabla_A$ on $\partial M_\rho \times [0, 1]$ given by $A := A^t_\rho + 0 \cdot \mathrm{d}t$. Clearly, this connection is flat, i.e., $F_A = 0$. The Chern–Simons theorem [1] implies that

$$\tau_{\partial M_\rho}(A_\rho) - \tau_{\partial M_\rho}(B_\rho) = -\frac{1}{8\pi^2}\int\limits_{\partial M_\rho \times [0,1]} \mathrm{tr}(F_A \wedge F_A) = 0,$$

concluding the proof. □

This lemma is used in the estimates on p. 293 and p. 299 in [2]. In these estimates, the original (incorrect) $L^2$ norm of the $\mathfrak{su}(2)$ valued 1-form $A_\rho - \Gamma_\rho$ should be replaced simply by its $L^2_{1,\Gamma_\rho}$ norm dictated by the corrected Lemma 2.1 presented here. This replacement is only of technical nature and *does not effect any of the main results* in [2].

Finally, we would like to thank U. Bunke for pointing out this technical gap.

## References

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