



Mathematical model to assess the control of *Aedes aegypti* mosquitoes by the sterile insect technique

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Abstract

We propose a mathematical model to assess the effects of irradiated (or transgenic) male insects introduction in a previously infested region. The release of sterile male insects aims to displace gradually the natural (wild) insect from the habitat. We discuss the suitability of this release technique when applied to peri-domestically adapted *Aedes aegypti* mosquitoes which are transmitters of Yellow Fever and Dengue disease.

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1. Introduction

Diverse disciplines of science as radiation biology, chemistry, ecology, molecular biology, genetics and entomology have contributed to the control of insect pests. In this context, Knipling [1] had conceived an approach to insect control in which the natural reproductive processes of insects are disrupted by the use of mutagens such as gamma radiation thus rendering the insects

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sterile. These sterile insects are then released into the environment in very large numbers in order to mate with the native insects that are present in the environment. A native female that mates with a sterile male will produce eggs, but the eggs will not hatch (the same effect will occur for the reciprocal cross). If there is a sufficiently high number of sterile insects then most of the crosses are sterile, and as time goes on, the number of native insects decreases and the ratio of sterile to normal insects increases, thus driving the native population to extinction. This insect control method is now known as the sterile insect release method (SIT) [2].

Recently, techniques like genetic control have been considered, which consist of genetic manipulation to produce transgenic insects in order to result in sterility or lethal (or deleterious) genes [2]. The introduction of deleterious genes into the population via sperm that actually fertilizes the egg has the advantage of being independent of the number of matings. This in turn avoids the massive release of sterile males according to SIT.

In the traditional insecticide control, the amount of applied pesticides and the subsequent cost are generally proportional to the area treated and not to the size of the population. However, the damage done by a pest is proportional to the number of individual pest per unit area. Thus, the cost/benefit ratio of insecticidal control increases as the size of the pest population increases. But, in sterile male release programs, the number of sterile male released and their cost are proportional to the size of the population. Since the cost/benefit ratio decreases with a decrease in size of the pest population, it can be wise to integrated insecticidal methods with sterile male technique.

Sterile male techniques were first used successfully in 1958 in Florida to control Screwworm fly (*Cochliomya omnivorax*) [3,4]. About 50 million flies of both sexes were released per week over an 18 months period, in a total of 2 billion flies over 85000 square mile area. The pest was eradicated after this period of application, in which 40 tons of ground meat were required each week and 20 aircraft to release the sterile flies. The total cost was about US\$ 10000000.00.

Although the screwworm eradication program is the most successful use of SIT, other insect species have been subjected to the release of sterile insects with varying success. Some examples are screwworm fly in USA, Mexico and Libya; Mediterranean Fruit Fly (*Ceratitis capitata Wiedemann*) in USA and Mexico; Melon Fly (*Dacus cucurbitae Coquillett*) in Japan and Taiwan; Pink Bollworm (*Pectinophora gossypiella Saunders*) in USA; Tsetse Fly (*Glossina species*) in Tanzania, Zimbabwe and Upper Volta; Boll Weevil (*Anthonomus grandis Boheman*) in Southeastern USA; Mexican Fruit Fly (*Anastrepha ludens Loew*) in USA and Mexico; Gypsy Moth (*Lymantria dispar* Linnaeus) in USA and Canada [2].

The reasons for success in some cases and little or not success in others are due to limitations on the technique. These limitations include lack of competitive ability of sterile insects, lack of complete sterilization, insufficient sterile population, inadequate spraying, multiple matings, etc. A number of mathematical models have been done to assist the effectiveness of the SIT (see, e.g., [1,5–10]). Some of these models contemplate combination of SIT with other control measures as pesticides [11] or release of parasitoids [12].

Much research was carried out about 30 years ago, especially in India and El Salvador, on the application of SIT to mosquitoes. Unfortunately this technique virtually ends in the mid-1970s, not because the method was a technical failure, but because of political problems in India and intensifying civil wars in Central America [13]. There is now a revival of interest especially in the use of transgenesis to improve sex separation so that only non-biting males are released

and to ensure that their female progeny die without the need for radiation or chemosterilization [14]. Therefore, with some improvements through the use of currently available transgenic technologies, SIT could become a mainstay for public health control of specific vector-borne diseases.

In this paper we are concerned about the application of the SIT for the control of *Aedes aegypti* mosquitoes, which is the principal transmissor of Yellow Fever and Dengue disease. This pest species have more than one life stage, which is known that complicates the outcome of a sterile release program [15]. In order to shed further light on the effectiveness of the technique for control of mosquitoes we propose a general model that incorporates two life stages of the pest population. A question to be addressed is how the recruitment of the sterile males affects the efficiency of the SIT. We also consider two issues of mosquito behavior that are specially relevant to SIT, namely, (a) mating competitiveness of artificially reared sterilized males; (b) dispersal ability to ensure that sterile males are released near enough to all emerging wild females so that they have a fair chance of mating with them.

2. The model

Aiming the description of the dynamics of biological control, the life cycle of an insect is divided in two stages: the immature (eggs, larvae and pupae) and the adult one. In the case of mosquitoes the immature phase occurs in water. We denote by A the population size of the immature phase of the insect at time t . For the adult form we consider the following compartments: females before mating (singles), I ; mating fertilized females, F ; mating unfertilized females, U ; and male insects, M . The population size of sterile (irradiated or transgenic) insects at time t is denoted by M_T .

The per capita mortality rates of the immature form, unmating females, mating fertilized females, mating unfertilized females, natural (or wild) and sterile male insects are denoted by μ_A , μ_I , μ_F , μ_U , μ_M and μ_T , respectively.

The net oviposition rate per female insect is proportional to their density, but it is also regulated by a carrying capacity effect depending on the occupation of the available breeder sites. In this model we assume that the per capita oviposition rate is given by $\phi(1 - \frac{A}{C})$, where C is the carrying capacity related to the amount of available nutrients and space, and ϕ is the intrinsic oviposition rate. The immature population becomes adult insects at a per capita rate γ ; a proportion r of such are females and $1 - r$, males.

Flows from I to F and U compartments depend mainly on the number of encounters of females with native and sterile males, and on the correspondingly mating rates. Here we assume that the probability of an encounter of a female with natural insects is given by $\frac{M}{M+M_T}$. Then the per capita rate at which female insects are fertilized is $\frac{\beta M}{M+M_T}$, where β is the mating rate of natural insects.

Since sterile (irradiated or transgenic) insects are placed artificially, it is natural to think that the probability of an encounter of a sterile male with a female depends not only on the number of such males ($\frac{M_T}{M+M_T}$), but also on how far they are placed from the breeding sites. We will assume that this net probability is given by $\frac{pM_T}{M+M_T}$, where the parameter p , with $0 \leq p \leq 1$, is the proportion of sterile insects that are sprayed in the adequate places. Also, the effective fertilization during the mating could be diminished due to the sterilization, which leads us to assume that the effective mating rate of sterile insects is given by $q\beta$, with $0 \leq q \leq 1$. Putting together the assumptions

above we get that $\frac{\beta_T M_T}{M + M_T}$ is the per capita rate at which female insects are fecundated with sterile sperm, where $\beta_T = pq\beta$. In some extent, the parameter p is related to the effectiveness of sterile male introduction regarded to the spatial distribution of female insects, while q can be thought as a physiological modification induced by the sterilization technique. The parameter p plays an important role in the case of insects whose reproduction depends strongly on the spatially non-homogeneous distribution of breeding sites, like mosquitoes *A. aegypti*.

Finally, we assume that sterile insects are recluded and sprayed at a constant rate denoted by α . According to the assumptions above the model is given by

$$\begin{aligned} A' &= \phi \left(1 - \frac{A}{C} \right) F - (\gamma + \mu_A)A, \\ I' &= r\gamma A - \frac{\beta MI}{M + M_T} - \frac{\beta_T M_T I}{M + M_T} - \mu_I I, \\ F' &= \frac{\beta MI}{M + M_T} - \mu_F F, \\ M' &= (1 - r)\gamma A - \mu_M M, \\ M_T' &= \alpha - \mu_T M_T \end{aligned} \tag{1}$$

and the remaining decoupled equation for mating unfertilized females is

$$U' = \frac{\beta_T M_T I}{M + M_T} - \mu_U U.$$

3. Equilibrium points

The population of sterile insects approaches the equilibrium $\frac{\alpha}{\mu_T}$, independently of the initial conditions. Then, system (1) has a trivial equilibrium $P_0 = \left(0, 0, 0, 0, \frac{\alpha}{\mu_T} \right)$, with $\bar{U} = 0$, corresponding to the state where natural insects are absent, and there is only a constant population of sterile insects. The non-trivial steady states $\left(\bar{A}, \bar{I}, \bar{F}, \bar{M}, \frac{\alpha}{\mu_T} \right)$ satisfy the following relations:

$$\begin{aligned} \bar{I} &= \frac{r\gamma \bar{A} \left(\bar{M} + \frac{\alpha}{\mu_T} \right)}{(\mu_I + \beta)\bar{M} + (\mu_I + \beta_T) \frac{\alpha}{\mu_T}}, \\ \bar{F} &= \frac{(\gamma + \mu_A)C\bar{A}}{\phi(C - \bar{A})}, \\ \bar{M} &= \frac{(1 - r)\gamma \bar{A}}{\mu_M}, \end{aligned} \tag{2}$$

where \bar{A} is a solution of the second degree equation

$$p(A) = aA^2 + bA + c = 0, \tag{3}$$

with coefficients

$$a = \frac{1}{C} \frac{\phi r \gamma \beta}{(\gamma + \mu_A)(\beta + \mu_I)\mu_F},$$

$$b = 1 - \frac{\phi r \gamma \beta}{(\gamma + \mu_A)(\beta + \mu_I)\mu_F},$$

$$c = \frac{(\beta_T + \mu_I)\mu_M \alpha}{(\beta + \mu_I)(1 - r)\gamma \mu_T}.$$

The remaining decoupled mating unfertilized females is given by

$$\bar{U} = \frac{\alpha \beta_T \bar{I}}{\mu_U(\mu_T \bar{M} + \alpha)}.$$

From the expression for \bar{F} it follows that the positive non-trivial equilibrium solutions must satisfy $0 < \bar{A} < C$. Now, at the endpoints of this interval

$$p(0) = c > 0,$$

$$p(C) = C + c > 0,$$

$$\frac{dp}{dA}(C) = \frac{\phi r \gamma \beta}{(\mu_A + \gamma)(\beta + \mu_I)\mu_F} + 1 > 0.$$

It follows that $p(A)$ has one or two roots inside the interval $(0, C)$ if and only if (i) $\frac{dp}{dA}(0) < 0$ and (ii) $b^2 - 4ac > 0$. Let us define

$$R = \frac{\phi r \gamma \beta}{(\mu_A + \gamma)(\beta + \mu_I)\mu_F} \tag{4}$$

and

$$S = \frac{(\beta_T + \mu_I)\mu_M \alpha}{(\beta + \mu_I)(1 - r)\gamma C \mu_T}. \tag{5}$$

Then, Eq. (3) can be written as

$$p(A) = \frac{R}{C} A^2 - (R - 1)A + CS = 0 \tag{6}$$

and the conditions for biological existence of the non-trivial equilibria become

$$R > 1 \tag{7}$$

and

$$S \leq \frac{(R - 1)^2}{4R} \equiv S^c. \tag{8}$$

When both conditions hold, the solutions are given by

$$\bar{A}_- = \frac{(R - 1)}{2R} C \left[1 - \sqrt{1 - \frac{4RS}{(R - 1)^2}} \right] \tag{9}$$

and

$$\bar{A}_+ = \frac{(R - 1)}{2R} C \left[1 + \sqrt{1 - \frac{4RS}{(R - 1)^2}} \right]. \tag{10}$$

Therefore, under conditions (7) and (8), system (1) has two positive equilibria P_{1-} and P_{1+} , correspondingly to \bar{A}_- and \bar{A}_+ . If equality holds in (8) then P_{1-} and P_{1+} collapse to an equilibrium P_1 with $\bar{A} = \frac{(R^*-1)}{2R^*} C$, which provides minimum R^* that guarantees the existence of the non-trivial equilibria given by

$$R^* = (1 + 2S) \left[1 + \sqrt{1 - \frac{1}{(1 + 2S)^2}} \right]. \tag{11}$$

Notice that $R^* > 1$.

The parameter R can be interpreted biologically as follows: since $\frac{1}{\gamma + \mu_A}$ is the average time of survival of the immature phase of the insect, $\frac{1}{\gamma}$ is the average time of its permanence as such, and r is the fraction of immature forms that become females, then $\frac{r\gamma}{\gamma + \mu_A}$ is the probability that an egg will succeed to become a female insect. Arguing in the same way, $\frac{\beta}{\beta + \mu_I}$ is the probability that a female will be fertilized, and finally $\frac{\phi}{\mu_F}$ is the average number of eggs oviposited by each fertilized female. Thus, the product of these three quantities, which is equal to R , is the average number of secondary female insects produced by a single female insect. In demographic terms R is the *basic offspring number* of insect population (equivalent to basic reproductive number in the epidemiological context). For natural (or wild) insects to maintain in nature, condition $R > 1$ is necessary. However, it is reasonable to assume that, in the presence of sterile insects, this could not be a sufficient condition since a proportion of the females are not fertilized.

The expression R can be rewritten as $R = \frac{\phi}{\phi^{th}}$, where

$$\phi^{th} = \frac{(\mu_a + \gamma)(\beta + \mu_I)\mu_F}{r\gamma\beta},$$

is the threshold oviposition rate. Hence, for $\phi > \phi^{th}$ we have the infestation of mosquito population at a level that depends on ϕ ; otherwise, we have the mosquito population going to extinction. The model considers a carrying capacity, which limits the mosquito population. Hence, we have the limiting threshold oviposition rate $\phi^{th} = \frac{(\mu_a + \gamma)\mu_F}{r\gamma}$ for $\beta \rightarrow \infty$. Note that when $\beta \rightarrow 0$, we have $\phi^{th} \rightarrow \infty$ and $R < 1$ for finite oviposition rate.

The parameter S can be written as the ratio between s_1 and s_2 , where

$$s_1 = (\beta_T + \mu_I) \frac{r\gamma C}{\mu_I} \frac{\alpha}{\mu_T} \quad \text{and} \quad s_2 = (\beta + \mu_I) \frac{r\gamma C}{\mu_I} \frac{(1 - r)\gamma C}{\mu_M}.$$

The quantities s_1 and s_2 represent the maximal mating rates among female insects and, respectively, sterile and natural male insects; then S measures the number of mated but not fertilized female insects with respect to the fertilized ones. Also, when $\beta = \beta_T$, S measures the ratio between the number of sterile males and the number of natural insects in equilibrium. If S is sufficiently high ($S \geq S^c$), the next generation of wild insects would be lower than the actual one since a proportion of eggs would not hatch. If sterile male insects are sprayed for a long period of time, this pattern would drive the natural insect population to zero.

Therefore, whenever $S \geq S^c$, we have the displacement of the natural insects. The critical number of mated but not fertilized insects with respect to the fertilized ones S^c , increases with the basic offspring number R , with bounds $[0, \frac{R}{4}]$, for $R = 1$ and $R \gg 1$ (for $\phi \gg 1$), respectively. In the case of mosquito population, the value of R has great variations during the year, being much more bigger during the rainy season than in the dry season. Then, the number of mosquitoes that should be sprayed, and consequently the cost of SIT will depend heavily upon the time of the year when this control technique is applied.

4. Stability of equilibria

In this section we analyze conditions for stability of the equilibrium points. For this, we calculate the eigenvalues regarded to the Jacobian of system (1), given by

$$J = \begin{pmatrix} -\frac{\phi \bar{F}}{C} - (\mu_A + \gamma) & 0 & \phi \left(1 - \frac{\bar{A}}{C}\right) & 0 & 0 \\ r\gamma & -\mu_I - \frac{\beta \bar{M} + \beta_T \bar{M}_T}{\bar{M} + \bar{M}_T} & 0 & \frac{(\beta_T - \beta) \bar{M}_T \bar{I}}{(\bar{M} + \bar{M}_T)^2} & \frac{(\beta - \beta_T) \bar{M} \bar{I}}{(\bar{M} + \bar{M}_T)^2} \\ 0 & \frac{\beta \bar{M}}{\bar{M} + \bar{M}_T} & -\mu_F & \frac{\beta \bar{M}_T \bar{I}}{(\bar{M} + \bar{M}_T)^2} & -\frac{\beta \bar{M} \bar{I}}{(\bar{M} + \bar{M}_T)^2} \\ (1 - r)\gamma & 0 & 0 & -\mu_M & 0 \\ 0 & 0 & 0 & 0 & -\mu_T \end{pmatrix}, \tag{12}$$

evaluated at the equilibrium points. The eigenvalues of J at the equilibrium $P_0 = (0, 0, 0, 0, \alpha/\mu_T)$ are $-(\gamma + \mu_A)$, $-(\beta + \mu_I)$, $-\mu_F$, $-\mu_M$, and $-\mu_T$; therefore P_0 is always locally asymptotically stable. Global stability can be proved for $R \leq 1$ using the function $V : R_+^5 \rightarrow R$ given by

$$V = \frac{r\gamma}{\mu_A + \gamma} A + I + \frac{\phi r\gamma}{(\mu_A + \gamma)\mu_F} F, \tag{13}$$

whose orbital derivative

$$\dot{V} = - \left[\frac{r\gamma\phi}{(\mu_A + \gamma)C} \right] AF - \frac{(\beta + \mu_I)}{\bar{M} + \bar{M}_T} (1 - R)IM - \frac{(\beta_T + \mu_I)IM_T}{\bar{M} + \bar{M}_T}, \tag{14}$$

is less or equal to zero for $R \leq 1$. From inspection of system (1) it can be seen that the maximal invariant set contained in $\dot{V} = 0$ is P_0 . Then, from La-Salle Lyapunov Theorem [16], P_0 is globally asymptotically stable for $R \leq 1$.

Now, we will analyze the stability of the equilibria P_{1-} and P_{1+} . For these points it is clear that $-\mu_T$ is an eigenvalue of the Jacobian. To obtain the characteristic equation for the other four eigenvalues, we use the identities

$$\begin{aligned} \frac{\phi \bar{F}}{C} + \mu_A + \gamma &= (\mu_A + \gamma) \frac{C}{C - \bar{A}}, \\ r\gamma\phi \left(1 - \frac{\bar{A}}{C}\right) \frac{\beta \bar{M}}{\bar{M} + \bar{M}_T} &= \mu_F (\mu_A + \gamma) \left(\mu_I + \frac{\beta \bar{M} + \beta_T \bar{M}_T}{\bar{M} + \bar{M}_T}\right), \\ (1 - r)\gamma\phi \left(1 - \frac{\bar{A}}{C}\right) \frac{\beta \bar{M}_T \bar{I}}{(\bar{M} + \bar{M}_T)^2} &= \frac{\mu_F \mu_M (\mu_A + \gamma) \bar{M}_T}{\bar{M} + \bar{M}_T}, \end{aligned}$$

derived from system (1) in equilibrium. After some manipulation it can be seen that the eigenvalues are the roots of the polynomial

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4, \tag{15}$$

with

$$\begin{aligned} a_1 &= (\mu_A + \gamma) \frac{C}{C - \bar{A}} + \mu_I + \frac{\beta\bar{M} + \beta_T\bar{M}_T}{\bar{M} + \bar{M}_T} + \mu_F + \mu_M, \\ a_2 &= (\mu_A + \gamma) \left(\frac{C}{C - \bar{A}} \right) \left(\mu_I + \frac{\beta\bar{M} + \beta_T\bar{M}_T}{\bar{M} + \bar{M}_T} + \mu_F + \mu_M \right) \\ &\quad + \left(\mu_I + \frac{\beta\bar{M} + \beta_T\bar{M}_T}{\bar{M} + \bar{M}_T} \right) (\mu_F + \mu_M) + \mu_F\mu_M, \\ a_3 &= \left[\mu_I + \frac{\beta\bar{M} + \beta_T\bar{M}_T}{\bar{M} + \bar{M}_T} \right] \left[\mu_F(\mu_A + \gamma) \left(\frac{\bar{A}}{C - \bar{A}} \right) + \mu_M(\mu_A + \gamma) \left(\frac{C}{C - \bar{A}} \right) + \mu_F\mu_M \right] \\ &\quad + (\mu_A + \gamma)\mu_F\mu_M \left[\frac{C\bar{M} + \bar{M}_T\bar{A}}{(C - \bar{A})(\bar{M} + \bar{M}_T)} \right], \\ a_4 &= \frac{\mu_F\mu_M(\gamma + \mu_A)(\beta_T + \mu_I)\alpha}{(\bar{M} + \bar{M}_T)(C - \bar{A})\mu_TCS} (\bar{A}^2 + 2CS\bar{A} - C^2S). \end{aligned}$$

By the Routh–Hurwitz criteria, the roots of a polynomial of order four have negative real parts if and only if $a_i > 0$, with $i = 1, \dots, 4$, and $(a_1a_2 - a_3)a_3 > a_1^2a_4$. It is clear that for the polynomial (15) the coefficients a_1 , a_2 and a_3 are positive. Moreover, after some tedious calculations, it can be seen that the last condition is fulfilled for all positive \bar{A} . Therefore, the stability of both P_{1-} and P_{1+} is given by the sign of the coefficient a_4 , which is positive if and only if

$$s(\bar{A}) = \bar{A}^2 + 2CS\bar{A} - C^2S, \tag{16}$$

is greater than zero. Notice that $s(\bar{A})$ has a unique positive root $\bar{A} = A^* = CS\left(-1 + \sqrt{1 + \frac{1}{S}}\right)$ and, from (8), it can be seen that it satisfies the inequality

$$A^* > \frac{2CS}{R - 1}. \tag{17}$$

Now, we evaluate the polynomial (6) at A^* . Using (16), and inequalities (8) and (17), we obtain

$$\begin{aligned} p(A^*) &= \frac{R}{C}(-2CSA^* + C^2S) - (R - 1)A^* + CS = -(2RS + R - 1)A^* + (R + 1)CS \\ &< -(2RS + R - 1) \frac{2CS}{R - 1} + (R + 1)CS = -\frac{4RCS^2}{R - 1} + (R - 1)CS \\ &= (R - 1)CS \left(\frac{4RS}{(R - 1)^2} - 1 \right) < 0, \end{aligned}$$

according to the biological existence conditions (7) and (8). The inequality $p(A^*) < 0$ implies $\bar{A}_- < A^* < \bar{A}_+$ due to the positively defined coefficient of A^2 . Since $s(\bar{A})$ is such that $s(\bar{A}) < 0$

for $\bar{A} < A^*$ and $s(\bar{A}) > 0$ for $\bar{A} > A^*$, we have $a_4 < 0$ for \bar{A}_- and $a_4 > 0$ for \bar{A}_+ . Therefore P_{1-} is always unstable and P_{1+} , locally asymptotically stable.

We summarize the above in the following:

Theorem 1. *The equilibrium $P_0 = (0, 0, 0, 0, \frac{\alpha}{\mu_T})$ of the system (1) is always stable. When $R > 1$ and $\frac{(R-1)^2}{4RS} > 1$, the non-trivial equilibria, P_{1-} and P_{1+} , are feasible. In this case P_{1-} is always unstable and P_{1+} , stable.*

According to Theorem 1, for a fixed $R > 1$, if S is above $S^c = \frac{(R-1)^2}{4R}$ then it is possible to control insects by sterile male release, independently of their initial population size.

Let us discuss the above results in terms of the equilibrium points and their respective stability regions. In order to do this we analyze the number of immature phase in equilibrium \bar{A} as a function of the basic offspring number R and the recruitment rate α .

In the bifurcation diagram given by Fig. 1, the trivial equilibrium P_0 is represented by the R -axis. The stability of this point is global for $0 \leq R < R^*$, and local for $R > R^*$, where R^* is given by Eq. (11). When $R = R^*$, the turning equilibrium point P_1 appears, which is given by $P_1 = (\bar{A}, \bar{I}, \bar{F}, \bar{M}, \frac{\alpha}{\mu_T})$, where $\bar{A} = \frac{(R^*-1)}{2R^*}C$, and \bar{I}, \bar{F} and \bar{M} are obtained from Eq. (2) substituting \bar{A} above. For $R > R^*$ and $\alpha > 0$, P_1 bifurcates to the non-trivial equilibrium points P_{1-} and P_{1+} , which are unstable and locally stable, respectively. We call R^* the threshold value since it separates the region where we have only sterile insects ($R < R^*$) from the region where natural and sterile mosquitoes coexist at two different levels ($R > R^*$).

The threshold value R^* depends on S , which is a linear function of α , according to Eq. (5), with $S(0) = 0$ and $S(\alpha \rightarrow \infty) \rightarrow \infty$. Therefore, the bounds for the threshold value are $R^*(0) = 1$ and $R^*(S \rightarrow \infty) \rightarrow \infty$.

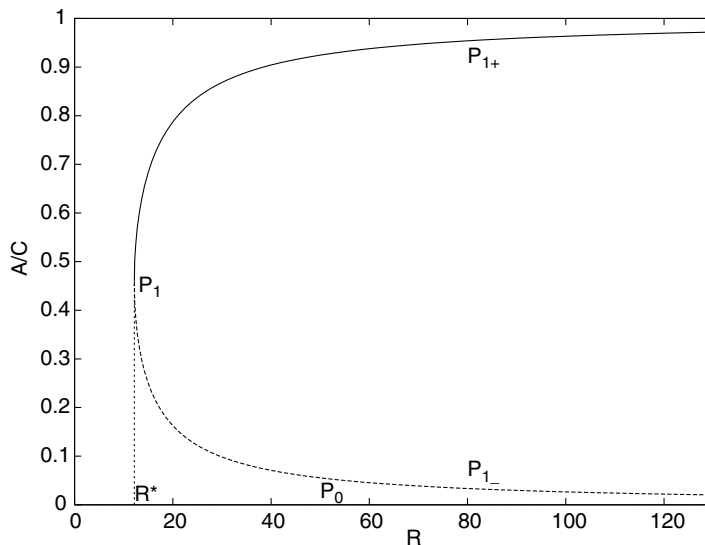


Fig. 1. Bifurcation diagram of system (1) with respect to R . The value $R^* = 12.16$ corresponds to $S = 2.56$ in Eq. (11).

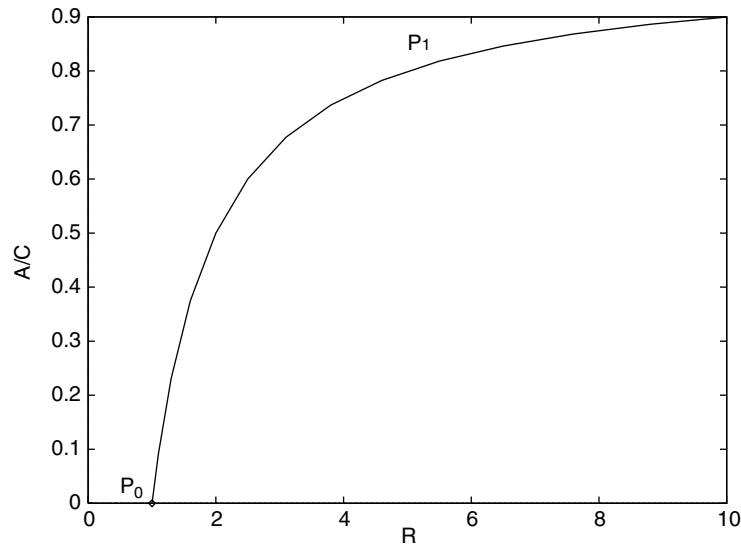


Fig. 2. Bifurcation diagram of system (1) with respect to R when $\alpha = 0$.

Notice that in the absence of sterile insects ($\alpha = 0$), the condition for existence of natural insects is $R > 1$. If this is not fulfilled, then we have the eradication of the insects, $P_0 = (0, 0, 0, 0, 0)$. At $R = 1$ we have the bifurcation value (see bifurcation diagram given in Fig. 2). However, when the sterile insects are released ($\alpha > 0$), the threshold value situates at $R = R^*(>1)$, indicating that one of the conditions to the existence of natural insects is furnished by increasing the basic offspring number. This is due to the decreasing in the mating fertilized females by the sterile male insects resulting in diminishing the net production of offsprings.

Now, let us consider the interval $R > R^*$. Recall that in this interval we have, besides the equilibrium point P_0 the equilibrium points P_{1-} and P_{1+} which are represented respectively by the lower and upper branches of the parabola in Fig. 1. We call break-point the unstable equilibrium point $P_{1-} = (\bar{A}_-, \bar{I}_-, \bar{F}_-, \bar{M}_-, \frac{\alpha}{\mu_T})$, where \bar{I}_-, \bar{F}_- and \bar{M}_- are obtained from Eq. (2) substituting \bar{A} by \bar{A}_- given by Eq. (9). The small root \bar{A}_- of Eq. (6) forms the decreasing branch of the polynomial's solution, which has the maximum value at $R = R^* (\bar{A} = \frac{R^*-1}{2R^*} C)$ and tends to the asymptote $\bar{A} = 0$, when $R \rightarrow \infty$. For $R > R^*$ this decreasing branch separates two attracting regions containing one of the equilibrium points P_0 and P_{1+} . In other words, we have a hyper-surface generated by the coordinates of the equilibrium point P_{1-} , e.g., $f(\bar{A}_-, \bar{I}_-, \bar{F}_-, \bar{M}_-, \frac{\alpha}{\mu_T}) = 0$, such that one of the equilibrium points P_0 and P_{1+} is attractor depending on the relative position of the initial conditions supplied to the dynamical system (1) with respect to the hyper-surface. The term ‘break-point’ was used by Macdonald to refer to the critical level for successful introduction of infection in terms of unstable equilibrium point [17].

For fixed $R > 1$, the bifurcation diagram with respect to α is illustrated in Fig. 3. When $\alpha = \alpha^*$, where α^* is the value for which $(R - 1)^2/4RS = 1$, or equivalently

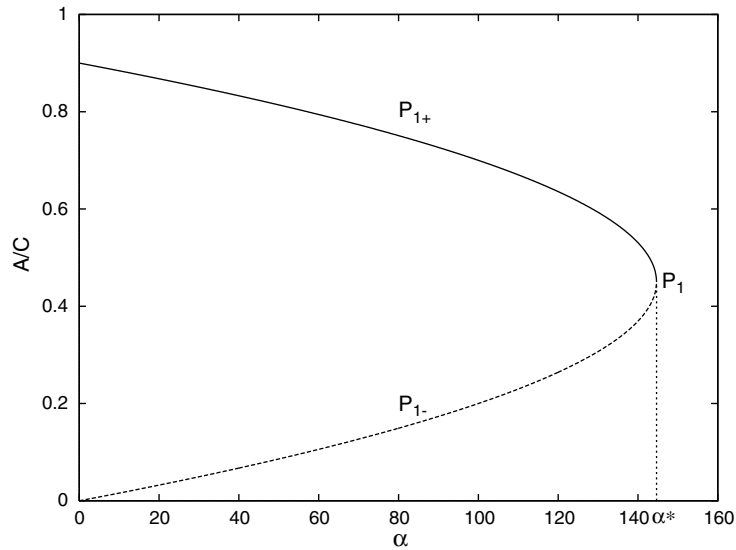


Fig. 3. Bifurcation diagram of system (1) with respect to α when $R > R^*$. The value of α^* in this diagram is ≈ 145 mosquitoes/day according to values listed in Table 1 of Section 5 and assuming $C = 600$.

$$\alpha^* = \frac{(\beta + \mu_I)(1 - r)\gamma\mu_T C (R - 1)^2}{(\beta_T + \mu_I)\mu_M 4R}. \tag{18}$$

P_{1-} and P_{1+} collapse to the equilibrium P_1 , and become infeasible for $\alpha > \alpha^*$. In the last case P_0 is globally asymptotically stable. Then, eradication of the pest is obtained beyond the threshold value α^* , disregarding the initial conditions. However, for α less than α^* control of the pest is not necessarily reached since the outcome will depend on the initial conditions. The same figure shows that for small α , the attracting region of P_{1+} increases.

5. Numerical results

In this section we present some numerical results for the model. The values of the parameters are given in Table 1. The vital variables ϕ , μ_A , μ_I , μ_F , μ_M , γ and r are according with values reported in the literature for *A. aegypti* mosquitoes (e.g., [18]), and will remain fixed otherwise indicated. We assume that $\frac{\beta_T}{\beta} = 0.7$, which means that sterile males loss 30% of the mating capacity due to physiological modifications and failures related to spatial distribution. Finally, we assume

Table 1
Parameter values for model (1)

ϕ	μ_A	μ_I	μ_F	μ_U	μ_M	μ_T	γ	r	β
5	0.05	0.05	0.05	0.05	0.1	0.1	0.075	0.5	1

Units are days⁻¹ except for r .

that the ratio $\frac{C}{\alpha/\mu_T}$ is 0.8. For these set of parameters $R = 21.16$, $S = 2.56$ and $S^c = 4.79$, which is greater than S . According to the analytical results this implies that P_0 and P_{1+} are locally stable, therefore success of sterile male insects release will depend strongly on the initial population size of the different insects classes.

The bifurcation diagram of Fig. 1 suggests that the unstable branch corresponding to $P_{1-} = (\bar{A}_-, \bar{I}_-, \bar{F}_-, \bar{M}_-, \frac{\alpha}{\mu_T})$ separates the regions of stability of P_0 and P_{1+} . More explicitly, initial conditions $(A(0), \bar{I}_-, \bar{F}_-, \bar{M}_-, \frac{\alpha}{\mu_T})$ with $A(0) < \bar{A}_-$ (resp. $A(0) > \bar{A}_-$) are in the basin of attraction of P_0 (resp. P_{1+}).

The above is confirmed by the numerical simulations as it is illustrated in Fig. 4. This figure shows the temporal course of the proportion A/C of the trajectories with initial conditions $(A(0), \bar{I}_-, \bar{F}_-, \bar{M}_-, \frac{\alpha}{\mu_T})$, where $A(0) = 0.999 \times \bar{A}_-$ and $A(0) = 1.001 \times \bar{A}_-$, respectively.

Numerical results show that the same behavior is obtained when we let \bar{A}_- fixed and move another coordinate of P_{1-} slightly above or below its value.

We also observe that the region of stability of P_0 decreases in a non-linear way with respect to R . This is illustrated in Figs. 5 and 6. In Fig. 5 we see that for $R = 21.16$ the initial conditions $(A(0), 0, 0, 0, \frac{\alpha}{\mu_T})$ with $A(0)/C < 0.4$ are in the basin of attraction of P_0 . When the size of R is duplicated ($R = 42.32$) only the trajectories with $A(0)/C < 0.1$ approach P_0 (see Fig. 6).

When S decreases the region of stability decreases even more faster as it is shown in Fig. 7. Here the value of R is as in Fig. 6 and $S = 1.28$.

Summarizing, numerical simulations indicate that when the equilibria P_0 and P_{1+} coexist, the stability region of the first one is rather small compared with the corresponding region of P_{1+} . In this situation, SIT would have a low chance to be successful, unless the immature population

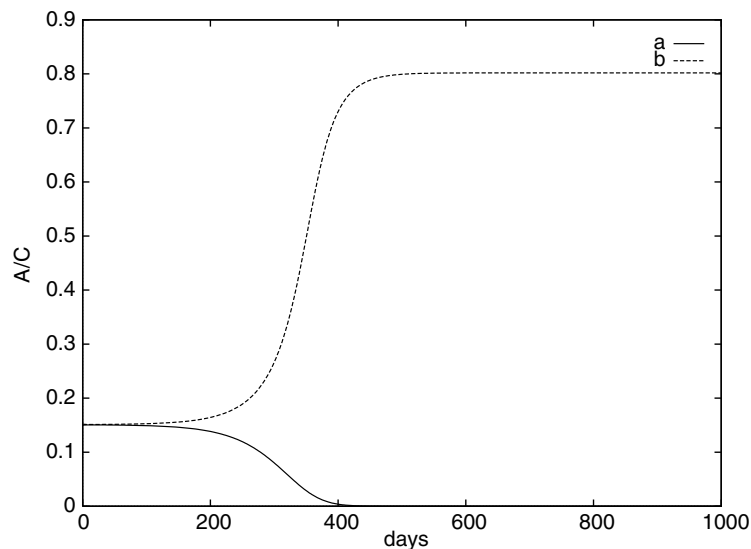


Fig. 4. Numerical simulations of system (1): (a) $A(0) = 0.999 \times \bar{A}_-$ and (b) $A(0) = 1.001 \times \bar{A}_-$. In this case $R = 21.16$, $S = 2.56$ and $S^c = 4.79$.

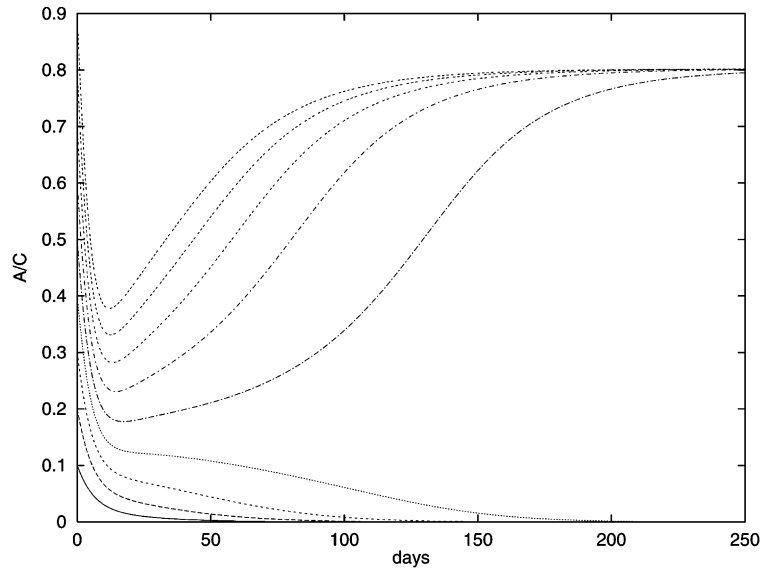


Fig. 5. Numerical simulations of system (1), for $R = 21.16$, $S = 2.56$ and $S^c = 4.79$. The initial conditions from bottom to top are $A(0)/C = 0.1, 0.2, \dots, 0.9$.

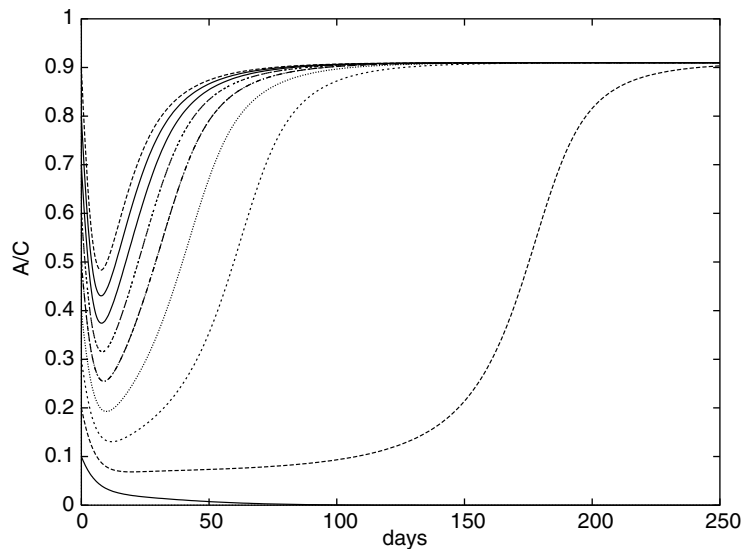


Fig. 6. Numerical simulations of system (1), with $\phi = 10 \text{ days}^{-1}$. In this case $R = 42$, $S = 2.56$ and $S^c = 10.06$. The initial conditions from bottom to top are $A(0)/C = 0.1, 0.2, \dots, 0.9$.

would be relatively small. Application of alternative controls to decrease the number of eggs, larvae and pupae (e.g., larvicides) would improve the effectiveness of SIT in this case.

Now, we analyze how mating capacity and dispersion affect the release of sterile males. For this, let $\bar{S} = \frac{\alpha/\mu_T}{(1-r)\gamma C/\mu_M}$ the ratio between sterile and natural male population in equilibrium, where for

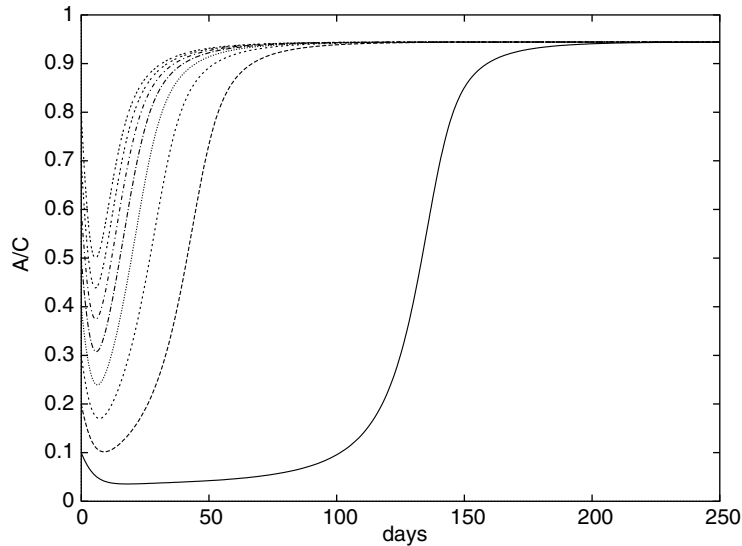


Fig. 7. Numerical simulations of system (1), with $\frac{C}{\alpha/\mu_T} = 1.2$, for $R = 42$, $S = 1.28$ and $S^c = 10.07$. The initial conditions from bottom to top are $A(0)/C = 0.1, 0.2, \dots, 0.9$.

the natural mosquitoes we made $\bar{A} = C$ (see third equation of (2)). From the definition of S given by (5) we have $\bar{S} = \frac{\beta + \mu_I}{pq\beta + \mu_I} S$ (recall that $\beta_T = pq\beta$, where q measures the percentage of reduction of mating capacity, and p , the ability of dispersion). In the best of the scenarios, we have $p = q = 1$, which implies $\bar{S} = S$. Taking a basic offspring number $R = 42.32$ (corresponding to an ovoposition of 10 eggs per day per female) the value of \bar{S} that assures completely effectiveness of SIT should be greater than $S^c = (R - 1)^2/4R = 10$, i.e., 10 sterile males for each wild male. As p or/and q decrement, the value of \bar{S} increases. Thus, for instance, if the mating capacity of sterile males is half of the one for wild males ($q = 0.5$), or the dispersion is $p = 0.5$, then \bar{S} should be greater than 19, i.e., the number of steriles for each natural mosquito should be almost doubled (here we assume the value of μ_I given above).

Another main question regarding the SIT control is the role of the introduction of wild mosquitoes in a region that is being controlled by the release of sterile males. To analyze this situation, let us suppose that sterile male release displaced the natural insects, so that system is in the state $P_0 = (0, 0, 0, 0, \frac{\alpha}{\mu_T})$. Additionally, let us consider the arrive of an invasion wave of wild mosquitoes as a non-zero initial condition in one of the mosquito stages. If the aquatic phase is transported, numerical simulations according with the parameters above indicate that the initial condition $A(0)$ should be above of $3.5 \times \bar{A}_-$ to initiate a re-colonization of wild mosquitoes that will approach the equilibrium P_{1+} . For fertilized females F , the condition $F(0) \geq 2.1 \times \bar{F}_-$ is necessary to initiate the re-infestation. On the other hand, waves of isolated mated and unfertilized females I as well as natural males M cannot re-invade. Then, as it was expected, the introduction of a sufficiently high number of fertilized females could contribute to the failure of SIT control.

6. Conclusion

In this paper we formulated a model to assess the effectiveness of the sterile insect release technique (SIT) applied to *A. aegypti* mosquito population. The results are given in terms of the basic offspring of natural population R , the ratio between the unfertile and fertile matings S , and the intrinsic rate of release of sterile males α .

Completely success of SIT can be achieved for R below the threshold $R^*(S)$ given by Eq. (11). Splitting this condition in terms of R , we have two cases. The first case, $R \leq 1$ does not need comments. However, when $R > 1$, SIT control effectively works when $S \geq S^c$, where S^c provides us the relation of sterile male insects released with respect to wild insects, that is, the displacement of natural mosquitoes can be obtained if the release rate of irradiated mosquitoes α is beyond the threshold α^* given by Eq. (18).

The model predicts that when R is above R^* , and α is below α^* , extinction of the wild insects depends on its initial population size. The stability diagrams given in Figs. 1 and 3, as well as the numerical simulations given in Section 3 suggest that the region of stability of P_0 is small compared with the corresponding region for P_{1+} , and decreases non-linearly when R increases or α decreases. In this case, it is possible that small perturbations of initial conditions in the stability region of P_0 drives the system to the coexistence equilibrium P_{1+} . One practical implication of this is that the introduction of fertile females in a region that it is under treatment by SIT could initiate a re-infestation of wild mosquitoes in such a way that the dynamics approach the coexistence equilibrium.

For mosquito population, mating competitiveness and dispersion of sterilized males are special relevant to SIT. These properties are modeled here via the parameters q and p . Field experiments using chemosterilized or chromosomally translocated and sex ratio-distorting *A. aegypti* have show moderately good mating competitiveness [19,20]. Adequate large releases (12–15 steriles for each wild male assuming a population with ovoposition rate equal to 10 eggs per day and $q = 0.8$) well mixed with isolated wild populations might have been expected to yield high levels of egg sterility. However, it seems that in the practice they frequently did not do so. This has been attributed to an influx of already mated females from outside the sterile male released area [13].

Dispersal ability is a major concern for the SIT to ensure that sterile males are released near enough to all breeding sites so that they have a chance of mating with wild females. In the model the reduction of chance of encounters is modeled by the factor p . Thus, p will have a high value for a highly dispersing species, meanwhile for a poorly dispersing one, $p \approx 0$. *A. aegypti* is considered a species having little dispersion and strongly localized around breeding sites, hence sterile males would need to be released at intervals of about 50 m along urban streets to find all the local females [21,22]. However, more recent studies [23] found that females can disperse over more than an 800 m radius. More studies should be done in order to clarify the grade of dispersion of such species.

The mathematical model presented in this paper do not include all factors affecting sterile releases. Some biological details are sacrificed in order to make a model mathematically tractable. Nevertheless, according to observations, one important point to consider is the immigration of females that have already had fertile matings and will lay fertile eggs nullifying the effect of sterile releases, which is left for a future work. Clarification of the role of immigrants in a population is important not only for the SIT, but also for assessing whether local efforts at larval control with insecticides of environmental management could have an impact on the adult population or whether they are likely to be swamped by immigration.

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