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Resumo. Stochastic volatility models consider the underlying security's volatility as a random process, governed by state variables such as the price level of the underlying security, the tendency of volatility to revert to some long-run mean value, and the variance of the volatility process itself, among others. Stochastic volatility models are one approach to resolve a shortcoming of the Black and Scholes model. In particular, these models assume that the underlying volatility is constant over the life of the derivative, and unaffected by the changes in the price level of the underlying security. However, these models cannot explain long-observed features of the implied volatility surface such as volatility smile and skew, which indicate that implied volatility does tend to vary with respect to strike price and expiry. By assuming that the volatility of the underlying price is a stochastic process rather than a constant, it becomes possible to model derivatives more accurately.

The Heston model introduced in 1993 has become one of the most widely used stochastic volatility models in the derivatives market. The model proposed by Heston take into account non-lognormal distribution of the assets returns, leverage effect, mean-reverting property of the volatility and it remains tractable. The price to pay for more

realistic models is the increased complexity of model calibration and the construction of hedging strategies.

Often, the estimation method becomes as crucial as the model itself. Research has shown that the implied parameters (i.e. those parameters that produce the correct vanilla option prices) and their time-series estimate counterparts are different. So one cannot just use empirical estimates for the parameters. This leads to a complication that plagues stochastic volatility models in general. A common solution is to find those parameters which produce the correct market prices of vanilla options. This is called an inverse problem, as we solve for the parameters indirectly through some implied structure. The most popular approach to solving this inverse problem is to minimize the error or discrepancy between model prices and market prices. More specifically, the squared differences between vanilla option market prices and that of the model are minimized over the parameter space.

We propose a probabilistic approach for estimating parameters of the Heston's stochastic volatility model from a set of observed option prices. Our approach is based on a stochastic optimization algorithm which generates a random sample from the set of global minima of the in-sample pricing error and allows for the existence of multiple global minima. We compare our algorithm with the adaptive simulated annealing (ASA).

With the parameters calibrated via our probabilistic algorithm, we need to develop hedging strategies for the Heston's stochastic volatility model considering general contingent claim. However, as it is well known, this market model is not complete. We then face with the problem of how valuing such claims and managing the risk we incur by buying or selling the claim. One of the earliest concepts for hedging and pricing in incomplete financial markets has been the quadratic criterion of local risk minimization introduced by Schweizer (1988) and Föllmer and Schweizer (1991). Existence of a locally risk minimizing strategy is related to the existence of the Föllmer-Schweizer decomposition, which can be viewed as an extension of the Kunita-Watanabe decomposition for semimartingales. For continuous semimartingale discounted price process, we can formulated the locally risk minimizing strategy based on the key fact that the Föllmer-Schweizer decomposition and the Kunita-Watanabe decomposition under the minimal martingale probability coincide. Based on the discretization scheme developed by Leão and Ohashi (2009), we develop one algorithm to provide locally risk minimizing hedges under Heston's stochastic volatility model for square integrable contingent claim. Finally, the empirical performance of locally risk minimizing delta hedges is tested using IBOV index over a two-and-half-year period.