On Picard curves over a finite field
(Saeed Tafazolian)

Let $\mathcal{C}$ be a projective, geometrically irreducible and non-singular algebraic curve defined over the finite field $\mathbb{F}_{q^2}$ with q^2 elements. We say that $\mathcal{C}$ is maximal over $\mathbb{F}_{q^2}$ if the number of its rational points attains the Hasse-Weil's upper bound:

$$\#\mathcal{C}(\mathbb{F}_{q^2}) = 1 + q^2 + 2gq.$$

Here we consider maximal Picard curves over a finite field with q^2 elements of characteristic $p > 3$. Let $\mathcal{C}$ be a Picard curve over $\mathbb{F}_{q^2}$. Then $\mathcal{C}$ can be defined by an affine equation of the form

$$y^3 = f(x),$$

where $f(x)$ is a polynomial over $\mathbb{F}_{q^2}$ of degree 4, without multiple roots.

The question which motivated this talk is: under what conditions on q is the Picard curve $\mathcal{C}$ given by the equation $y^3 = x^4 - x$ maximal over $\mathbb{F}_{q^2}$? In this regard we show

Theorem 1. *The smooth complete Picard curve $\mathcal{C}$ corresponding to $y^3 = x^4 - x$ is maximal over $\mathbb{F}_{q^2}$ if and only if $q \equiv -1 \ (mod\ 9)$.*

To prove the above theorem we need some information about supersingularity of the Picard curve $\mathcal{C}$. Here we study the Newton polygon of L-polynomial $L(t)$ associate to the curves

$$y^3 = x^4 - x,$$

defined over a finite field $\mathbb{F}_p$. Here we give a complete classification for the Newton polygon of $\mathcal{C}$. Indeed, our result completes the result which is given in [1].

References

[1] Y. Takizawa, Some remarks on the Picard curves over a finite field, *Math. Nachr.* **280** (2007), 802–811.