Seminario: Curvas Algébricas e temas afins

Campinas, 12 Agosto 2016

Title: On the a-number of curves in positive characteristic
Author: Pietro Speziali
Abstract: For an (algebraic, projective, absolutely irreducible) curve $\mathcal{X}$ defined over an algebraically closed field K of positive characteristic p, let $H^0(\mathcal{X}, \Omega_1)$ be the $g(\mathcal{X})$-dimensional vector space of holomorphic differentials on $\mathcal{X}$. The *Cartier operator* is the (unique) $1/p$-linear operator from $H^0(\mathcal{X}, \Omega_1)$ onto itself such that $C(\omega) = 0$ if and only if ω is exact and $C(\omega) = \omega$ if and only if ω is logarithmic. The *a-number* of $\mathcal{X}$ is the dimension $a(\mathcal{X})$ of the kernel of C, that is, the dimension of the space of holomorphic exact differentials.The a-number arises an invariant of the p-torsion group scheme $Jac(\mathcal{X})[p]$, but it may have deeper consequences on the structure of the Jacobian $\mathcal{J}(\mathcal{X})$ of $\mathcal{X}$. Also, by a classical result of Manin, the trace of m-th iterate of the Cartier operator is related to the number of $\mathbb{F}_{p^m}$-rational points of $\mathcal{X}$. The explicit computation of a-numbers appears to be a challenging task, thus making this problem of self-interest. Also, the explicit value of the a-number is known for just a few families of curves.

In this talk we survey the known results about the a-number, and in particular on the known strategies to compute it. Some new results on the a-numbers of two families of curves, namely the so-called Fermat type and Hurwitz type curves are also presented. We recall that a *Fermat curve* is the nonsingular plane curve $\mathcal{F}_n : X^n + Y^n + 1 = 0$ defined over the finite field $\mathbb{F}_p$ with $p \nmid n$, while a *Hurwitz curve* is the nonsingular plane curve $\mathcal{H}_n : X^nY + Y^n + X$ defined over $\mathbb{F}_p$ with $p \nmid n^2 - n + 1$.