

Free Malcev superalgebra on one odd generator and related superalgebras

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We consider the free Malcev superalgebra \mathcal{M} generated by one odd generator x and construct bases of \mathcal{M} and of its universal multiplicative envelope $\mathcal{R}(\mathcal{M})$. As a corollary, this gives us bases of spaces of all skew-symmetric elements and of all central skew-symmetric elements of free Malcev algebra of countable range.

Moreover, we construct a base of the Poisson Malcev superalgebra $\tilde{S}(\mathcal{M})$ associated with \mathcal{M} . In this way we obtain the elements which span the free alternative superalgebra \mathcal{A} generated by an odd element x . They can be considered as a natural candidate for a base of \mathcal{A} .

In fact, due to the computer program “Malcev”, we already know that the dimensions of the homogeneous components of $\tilde{S}(\mathcal{M})$ and \mathcal{A} of degree $n \leq 20$ coincide. It remains an open question whether it is true for all n . If the answer is positive, the superalgebra \mathcal{M} is special, i.e. the natural homomorphism $\pi : \mathcal{M} \rightarrow \mathcal{A}^-$, $\pi(x) = x$, is a monomorphism.