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Subgroups of Division Rings and Crossed Product Theorems

Let D be a finite-dimensional division ring of index m , and let G be a subgroup of the multiplicative group of D , such that G is soluble. It is a very old result (dating back to Suprunenko in the 1960s) that G has an abelian normal subgroup of index bounded by a (super-exponential) function of m . This has many consequences, including its recent use by Tignol and Mahdavi to show that certain finite-dimensional division rings containing metabelian subgroups are necessarily crossed products over maximal subfields.

We indicate how the above result can be generalized to the case where G is a finite extension of a soluble subgroup, and we give the best possible bound ($60m^2$) on the index of an abelian normal subgroup of G . We also show, by an example, that the question of whether D is a classical crossed product is essentially equivalent to long-standing conjectures on the structure of division algebras of prime index. Possible consequences for certain