VALUATIONS IN REAL ALGEBRA AND GEOMETRY

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We shall explain how valuations of arbitrary rank naturally show up in real algebra and geometry. They sometimes play a key role in the proof of theorems that do not even mention any valuation in their formulations. One such theorem is concerned with the characterization of "archimedean quadratic modules", i.e., a sequence of polynomials $h_1, ..., h_s \in \mathbb{R}[X_1, ...X_n]$ such that every polynomial f, strictly positive on the subset of $\mathbb{R}^n$ defined by the inequalities $h_1 \geq 0, ..., h_s \geq 0$, has a representation $f = \sigma_0 + \sigma_1 h_1 + ... + \sigma_s h_s$ where every σ_i is a sum of squares of real polynomials. In dimension two Eugenia Cabral has given a very efficient algorithm to decide which sequences $h_1, ..., h_s$ do define such a quadratic module (see her talk).