Free groups and involutions in the unit group of a group algebra

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Let $U(RG)$ denote the group of units of the group ring of a group G over a commutative ring with unity R. The study of the structure of $U(RG)$ is an interesting problem and it turns out that, in most cases, this group is quite large.

In the case when the coefficient ring is a field F or a ring of algebraic 1ntegers, the existence of free subgroups of rank 2 was studied and the explicit construction of such groups has been given by several authors.

At the light of these results, it is natural to ask which significant subgroups of the unit group are large in the sense that they still contain a free subgroup of rank 2 or, in other words, can one built the generators of the free group out of some special kind of units?. In this vein, Gonçalvez and Passman investigated the existence of free groups in the subgroup of unitary units with respect to the natural involution of FG induced by $g \mapsto g^{-1}$, for all $g \in G$.

We shall discuss the existence of free groups in another significant subgroup of $U(FG)$: the subgroup $U_2(FG)$ generated by all units of order 2 of FG. For an arbitrary torsion group G, we prove that if F is a non-absolute field and, in case $char(F) = p > 0$ G contains no p-elements, then $U_2(FG)$ contains a free group of rank 2 if and only if $U(FG)$ does so if this group is large, then the subgroup generated only by its involutions is already large.

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