Projective representations and Exel's theory

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In connection with studying of C^*-algebras R. Exel (R. Exel. *Partial actions of groups and actions of semigroups.* Proc. Amer. Math. Soc, **126**(1998), N12, 3481-3494., see also M. Dokuchaev, R. Exel, P. Piccione. *Partial representations and partial group algebras.* J. Algebra, **226** (2000), N1, 505-532.) introduced the notion of a partial linear representation (p.l.r.) of a group. By analogy with classic representation theory it is naturally to define partial projective representations (p.p.r.) of groups since they can be useful for description of p.l.r.'s.

Let G be a group, M the general matrix semigroup over a field F. *A partial projective representation of a group* G is a map $\Gamma : G \to M$ such that there are functions $\sigma, \tau : G \times G \to F$ for which

$$\begin{aligned}
\Gamma(x^{-1})\Gamma(xy) = 0 &\iff \sigma(x,y) = 0 \\
\Gamma(x^{-1})\Gamma(x)\Gamma(y) &= \Gamma(x^{-1})\Gamma(xy)\sigma(x,y) \\
\Gamma(xy)\Gamma(y^{-1}) = 0 &\iff \tau(x,y) = 0 \\
\Gamma(x)\Gamma(y)\Gamma(y^{-1}) &= \Gamma(xy)\Gamma(y^{-1})\tau(x,y) \\
\Gamma(1) &= E
\end{aligned}$$

Unlike the classic representations we have to use *two* factor systems σ and τ. Nevertheless,

Theorem 1 $\sigma = \tau$.

There is an analogy (and a close connection) between p.p.r.'s of groups and projective representations of semigroups (lasts were studied in B. V. Novikov. *On projective representations of semigroups.* Doklady AN USSR, 1979, N6, 474-478 (in Russian)., see also B. V. Novikov. *Semigroup cohomology and applications.* In: Algebra — Representation Theory, Kluwer, 2001, 219-234.). This enables to define the Schur multiplicator for p.p.r.'s of a group and to prove the next

Theorem 2 *The Schur multiplicator is a commutative inverse semigroup.*

However properties of p.p.r.'s essentially differ from ones of projective representations of semigroups. For example, the standard cohomological identity $\partial\sigma(x, y, z) = 0$ is not true for all elements of the group.

In the talk we also describe the idempotents of the Schur multiplicator.