# On recognizability of the simple groups PSL$(n, 2)$ through their spectrums

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**Abstract**

For a finite group $G$, we denote by $\pi_e(G)$ the set of all orders of elements in $G$, which has been recently called the *spectrum* of $G$. If $\Omega$ is a subset of $\mathbb{N}$ then $h(\Omega)$ denotes the number of pairwise non-isomorphic groups $G$ such that $\pi_e(G) = \Omega$. A group $G$ is called *k-recognizable* if $h(\pi_e(G)) = k < \infty$; otherwise $G$ is called *non-recognizable*. Also a 1-recognizable group is called a *characterizable* group. It has already been proved that the simple groups PSL$(n, 2)$ for $n = 3, 4, \ldots, 8, 11$ and 12 are characterizable. About simple groups PSL$(9, 2)$ and PSL$(10, 2)$, the problem still is open. Moreover, we have put forward the following conjecture:

CONJECTURE *For all positive integers $n \geq 3$, the simple groups PSL$(n, 2)$ are characterizable.*

In this paper, we first give a survey to all the simple groups PSL$(n, 2)$ which have already been characterized by their spectrums. We then continue to find some properties of the set $\pi_e(\text{PSL}(n, 2))$, whose proof is based on examining the structure of PSL$(n, 2)$ and on several arithmetic arguments, and also we calculate $\pi_e(\text{PSL}(13, 2))$. Finally, we characterize the projective special linear group PSL$(13, 2)$ using its spectrum.

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