

A Herstein's Theorem Extended to Generalized Higher Derivations

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Let R be an associative ring not necessarily with an identity element. A derivation (resp. Jordan derivation) d of R is an additive mapping $d: R \rightarrow R$ such that $d(ab) = d(a)b + ad(b)$, for every $a, b \in R$ (resp. $d(a^2) = d(a)a + ad(a)$, for every $a \in R$). As it is well-known, every derivation is a Jordan derivation and the converse is not true in general. If R is a 2-torsion-free semiprime ring, then by results by I.N. Herstein and M. Brešar, every Jordan derivation of R is a derivation. It turns out that every Jordan derivation of a 2-torsion-free ring is a Jordan triple derivation. We recall that an additive mapping $d: R \rightarrow R$ is said to be a Jordan triple derivation if $d(aba) = d(a)ba + ad(b)a + abd(a)$, for every $a, b \in R$.

Also, R. Awtar extended the Herstein's Theorem to Lie ideals by proving that if U is a Lie ideal of a prime ring R of $\text{char}(R) \neq 2$ such that $u^2 \in U$, for every $u \in U$, and $d: R \rightarrow R$ is an additive mapping such that $d|_U$ is a Jordan derivation of U into R , then $d|_U$ is a derivation of U into R .

Following B. Hvala, an additive mapping $F: R \rightarrow R$ is called a generalized derivation if there exists a derivation $d: R \rightarrow R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$. We call an additive mapping $F: R \rightarrow R$ a Jordan generalized derivation if there exists a derivation $d: R \rightarrow R$ such that $F(x^2) = F(x)x + xd(x)$ holds for all $x \in R$. In 2000, M. Ashraf and N. Rehman showed that in a 2-torsion-free ring R which has a commutator nonzero divisor, every Jordan generalized derivation on R is a generalized derivation.

On the other hand, higher derivations have been studied in many papers mainly in commutative rings, but also in non-commutative rings. M. Ferrero and C. Haetinger extended some of the above results to the higher derivations. In particular, they pointed out that every Jordan higher derivation in a 2-torsion-free semiprime ring is a higher derivation. Thus, it is natural to ask whether every Jordan generalized higher derivation on a ring R is a generalized higher derivation.

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