On towers of function fields over finite fields.

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The interest on the number of solutions of polynomial equations over finite fields has a long history in mathematics, going back at least to Gauss. When these polynomials define an one-dimensional object; i.e., define an algebraic curve or a function field, this research was crowned by the famous theorem of A.Weil, which is equivalent to the validity of the Riemann Hypothesis for the associated Zeta-function. Towers of function fields over finite fields (or of algebraic curves over finite fields) is fundamental to the study of the asymptotic behaviour of the number of rational places (or of rational points) with respect to the genus. It has deep connections with the asymptotic behaviour of linear codes over finite fields, implying in particular the existence of codes better than the Gilbert-Varshamov bound.

We are going to present an overview of this theory ,including the concepts: ramification locus, splitting locus, genus of a tower, limit of a tower, tame and wild towers, Kummer towers, Artin-Schreier towers, h-towers, dual towers, etc.

Several interesting examples illustrating these concepts will be presented.