

“On Lehmer’s problem on group varieties”

In this series of lectures, we shall describe modern approaches to the classical Lehmer’s problem, which suggests a lower bound of the height of an algebraic number in terms of its degree (provided it is not a root of unity). We shall first explain how the original problem is naturally formulated in terms of canonical heights of multiplicative groups or abelian varieties and recall what classical methods yield in the one dimensional case. In a second part, we shall expand to the general set up (a general subvariety of the group studied). Finally, we shall study relative approaches of the same problem (where the base field is itself variable) and explain how this relative situation often finds a natural geometric explanation. For most parts, we shall restrict proofs to the toric case to keep the exposition lighter and explain how the methods translate in the abelian case.