

Coset Space Approach for the Design of Geometrically Uniform Codes in Homogeneous Spaces

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In this paper we consider the use of surfaces or 2-dimensional manifolds as topological spaces in the design of geometrically uniform codes. The main idea behind this approach is to identify the surface topology (geometric and algebraic properties) associated with each block diagram forming the traditional model of a communication system, however starting this procedure with the graph associated with the discrete memoryless channel, DMC channel. Once the geometric and algebraic properties of the DMC channel are known, the design of the remaining block diagrams will have to be matched to these properties. The procedure employed to achieve this goal is based on the following steps: knowing the graph associated with a given discrete memoryless channel, 1) to determine the set of surfaces in which the graph is embedded; 2) to determine the fundamental group, and its subgroups, associated with each previous surface; and 3) to identify the regular tessellations which may be used in the design of signal constellations matched to subgroups of the fundamental group associated with the corresponding Riemann surfaces. As a consequence of the third step, we extend the concept of geometrically uniform codes, formerly employed in Euclidean spaces, to spaces with constant curvature, in particular to the hyperbolic spaces. We also show a characterization of the generalized coset codes through the concept of G -linear codes. Since the previous three steps may be extended to n -dimensional manifolds, we consider the performance analysis of the corresponding codes in Riemannian manifolds. To do that it is necessary to extend the concepts related to signal constellations, symbol error probability, and average energy of the signal constellation to the theory of differentiable manifolds. One of the important results coming out of this formalism is that the sectional curvature of the manifold is a relevant parameter in the design and in the performance analysis of signal constellations. As a consequence, we show that the best performance is achieved when the sectional curvature is constant and negative.

*This work has been supported by FAPESP under grant 02/07473-7, by CNPq under grant 311406-85, and by CAPES-PROCAD under grant 0121/01-0.