

On the bounds for the number of gaps of a Weierstrass semigroup of several points

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Let X be a nonsingular projective curve of genus g defined over a field K and such that K is the full field of constants of the function field $K(X)$ (so, e.g. if K is a finite field we are assuming that X is geometrically irreducible). Let P_1, \dots, P_n be points of X ; the Weierstrass semigroup associated to P_1, \dots, P_n is defined as

$$H(P_1, \dots, P_n) = H_n := \{(\alpha_1, \dots, \alpha_n) \in \mathbb{N}_0^n \mid \exists f \in K(X) \text{ with } \operatorname{div}_\infty(f) = \sum_{i=1}^n \alpha_i P_i\},$$

where by $\operatorname{div}_\infty(f)$ we mean the pole divisor of $f \in K(X)$. We will assume that P_1, \dots, P_n are rational points of X .

We call a n -tuple $(\alpha_1, \dots, \alpha_n) \in \mathbb{N}_0^n$ a *gap* if it is not an element of H_n , otherwise we call it a *non-gap*. From the Riemann-Roch theorem we get that the number of gaps is finite. It is well known that in the case where $n = 1$ the number of gaps is always equal to g , but if $n \geq 2$ this number may depend on the choice of the points, as has been observed by S. J. Kim, who started the systematic study of H_2 . In that same paper, he found a sharp lower bound for the number of gaps of H_n when $n = 2$; in the present talk we will show an extension of that result to any value of n , by proving that $\#G_n \geq \binom{n+g}{n} - 1$ for all $n \geq 2$ (here $G_n = G(P_1, \dots, P_n) := \mathbb{N}_0^n \setminus H(P_1, \dots, P_n)$ denotes the set of gaps of H_n). We will also show that this bound is attained “generically” on any curve X if the field of definition is algebraically closed and of characteristic zero. Finally, we will discuss some results on the upper bound for $\#G_n$.