Modular group algebras with maximal Lie nilpotency indices

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Let R be an associative algebra with identity. The ring R can be treated as a Lie algebra under the Lie product $[x, y] = xy - yx$, where $x, y \in R$. Set $[x_1, \ldots, x_n] = [[x_1, \ldots, x_{n-1}], x_n]$, where $x_1, \ldots, x_n \in R$. The *n-th lower Lie power* $R^{[n]}$ of R is the associative ideal generated by all Lie products $[x_1, ..., x_n]$, where $R^{[1]} = R$ and $x_1, ..., x_n \in R$. The *n-th upper Lie nilpotent power* $R^{(n)}$ of R is the associative ideal generated by all Lie products $[x, y]$, where $R^{(1)} = R$ and $x \in R^{(n-1)}$, $y \in R$.

The ring R is called *Lie nilpotent* if there exists m such that $R^{[m]} = 0$. The minimal integers m, n such that $R^{[m]} = 0$ and $R^{(n)} = 0$ are called *the lower Lie nilpotency indices* and *the upper Lie nilpotency indices* of R and they are denoted by $t_L(R)$ and $t^L(R)$, respectively.

Let $R = KG$ be the Lie nilpotent group algebra with $char(K) = p > 0$. If KG is Lie nilpotent then $t_L(KG) \leq t^L(KG) \leq |G'|+1$. Moreover, if $char(K) > 3$, then $t_L(KG) = t^L(KG)$ (see Bhandari, A. K.; Passi, I.B.S. *Lie Nilpotency indices of group algebras* Bull.London Math. Soc. **24** (1992), 68–70). But the question, when is $t_L(KG) = t^L(KG)$ for $char(K) = 2, 3$ is still open.

As A. Shalev (*The nilpotency class of the unit group of a modular group algebra. III,* Arch. Math. (Basel), **60** (2) (1993), 136–145) proved, if G is a finite p-group and $char(K) \geq 5$, then $t_L(KG) = |G'| + 1$ if and only if G' is cyclic. We proved the following:

Theorem(V. Bovdi, E.Spinelli) *Let KG be a Lie nilpotent group algebra with $char(K) = p > 0$. Then $t_L(KG) = |G'| + 1$ if and only if one of the following conditions holds:*

- *G' is cyclic;*
- *$p = 2$ and G' is noncyclic of order 4 and $\gamma_3(G) \neq 1$.*

Corollary *Let KG be a Lie nilpotent group algebra with $char(K) > 0$ such that $t_L(KG) = |G'| + 1$. Then $t_L(KG) = t^L(KG)$.*