Cohomology and subgroups of the Brauer group

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Let k be a field and $Br(k) \cong H^2(G_k, k_s^*)$ its Brauer group. The Schur subgroup $S(k)$ of $Br(k)$ is the subgroup consisting of classes that are represented by Schur algebras, namely k-central simple algebras that are spanned by a finite group of units. Equivalently a k-central simple algebra is Schur if it is the homomorphic image of a group algebra kG for some finite group G. The Schur group is trivial for fields of positive characteristic. For fields of characteristic zero $S(k)$ is rather "small". A theorem of Brauer and Witt provides a cohomological description of $S(k)$ namely, the subgroup that corresponds to the image of $H^2(Gal(k_{cyc}/k), \mu) \to H^2(G_k, k_s^*)$ where k_{cyc} is the maximal cyclotomic extension of k and μ is the group of all roots of unity. In analogy to the Schur group, $PS(k)$, the projective Schur group was defined by Lorenz and Opolka (1978). It consists of Brauer classes that are represented by k-central simple algebras spanned by a subgroup of units that is finite modulo its center. The symbol algebras are examples of projective Schur algebras. $PS(k)$ is much larger than $S(k)$. For instance, if k is a local or global field,$PS(k) = Br(k)$. If k contains all roots of unity $S(k) = 0$ whereas $PS(k) = Br(k)$ by the Merkurjev-Suslin theorem. It was conjectured that $PS(k) = Br(k)$ for any field but it turned out to be false (e.g. $k = Q(x)$). In the lecture I'll explain several (old and new) results and in particular I'll consider the question of representing $PS(k)$ via cohomology.