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Elliptic systems involving Schrödinger operators with vanishing potentials

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Abstract

We prove the existence of a bounded positive solution of the following elliptic system involving Schrödinger operators

$$\begin{cases} -\Delta u + V_1(x)u = \lambda\rho_1(x)(u+1)^r(v+1)^p & \text{in} \quad \mathbb{R}^N \\ -\Delta v + V_2(x)v = \mu\rho_2(x)(u+1)^q(v+1)^s & \text{in} \quad \mathbb{R}^N, \\ u(x), v(x) \to 0 & \text{as} \quad |x| \to \infty \end{cases}$$

where $p, q, r, s \geq 0$, V_i is a nonnegative vanishing potential, and ρ_i has the property (H) introduced by Brezis and Kamin [1]. As in that celebrated work we will prove that for every $R > 0$ there is a solution (u_R, v_R) defined on the ball of radius R centered at the origin. Then, we will show that this sequence of solutions tends to a bounded solution of the previous system when R tends to infinity. Furthermore, by imposing some restrictions on the powers p, q, r, s without additional hypotheses of integrability on the weights ρ_i, we obtain a second solution using variational methods. In this context we consider two particular cases: a gradient system and a Hamiltonian system. This work is motivated in part by [2] and [3].

Joint work with Denilson Pereira and Juan Arratia.

References

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