Regional Program MATH-AmSud
2020 Project Proposal (Research - Innovation)

Basic Form

A. General Information

A1	Project Title
	Geometric Structures and Moduli Spaces

A2	Acronym
	GS&MS

A3	Research domain
	Geometry and Topology: Algebraic and Differential Geometry.

A4	Project goals
	We propose a four-country collaboration to advance the understanding special geometric structures on differentiable manifolds and to contribute towards possible resolutions of three classical problems in algebraic geometry which have moduli spaces as a background. The project's goal is to consolidate a network of experts working on different aspects of Gemetry & Topology, in order to collaboratively contribute to the development of techniques and the advance of the theory by sharing the expertise of each research group. Among the participants of the present project, a significative number of binational collaborations between research groups already exists. Indeed, the proposal builds upon two concomitant CAPES-COFECUB French-Brazilian collaborations, "Moduli spaces in algebraic geometry and applications" (926/19) and "Special geometries and gauge theory" (898/18), and mobilises synergies between local networks of geometers in all countries, in order to attempt significant advances towards the understanding of special geometric structures and their connections to gauge theory. At the same time, it adds new research teams from Argentina, Chile and France to enlarge the existing networks. The motivation for putting up this Math-AmSud project is to strengthen such partnerships but also to consolidate them into a bigger framework of a four-sided network, enhancing the productiveness, the collaborations and facilitating the formation of new researchers. The collaborations will articulate around the Brazilian team led by Marcos Jardim and Henrique Sá Earp, both of Universidade Estadual de Campinas (Unicamp) and each involved at the heart of, respectively, the Algebraic Geometry and Differential Geometry branches of the project. This team will act as leader of the research groups and intermediate between the South American teams and members in France.

A5	Abstract
	This project proposes to advance in the research of several topics within the areas of differential and algebraic geometry and topology. The main objectives are (i) to understand special geometric structures on Riemannian manifolds, through their interplay with geometric flows and with complex algebraic geometry; and (ii) to focus on classical problems in algebraic geometry having moduli spaces as background. This proposal has both theoretical and example-oriented objectives, with, on the one hand, qualitative results and, on the other, the construction of explicit geometric structures.

A6	Scientific coordinators at each institution			
	South America A - Brazil		**South America B - Argentina**	
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A7	Other participating institutions

In South America

– Instituto de Matemática (IM), Universidade Federal do Rio de Janeiro (UFRJ), Brazil.

– Instituto de Matemática e Estatística (IME), Universidade Federal Fluminense (UFF), Brazil.

– Instituto de Ciências Exatas (ICEx), Universidade Federal de Minas Gerais (UFMG), Brazil.

– Departamento de Matemática (MTM), Universidade Federal de Santa Catarina (UFSC), Brazil.

In France

– Laboratoire de Mathématiques et de leurs interactions (LMAP-UMR 5142), Université de Pau et des Pays de l'Ardour (UPPA)

– Université de Bourgogne (UB), Institute de Mathématiques de Bourgogne (IMB-UMR 5584)

– Institut de Mathématiques de Toulouse (IMT-UMR 5219)

– Institut Fourier, Grenoble (IF-UMR 5582)

A8	List of expected participants (name and affiliation and status : junior, senior)

Argentinian Team

Senior: Adrián Andrada (UNC-CONICET), María Laura Barberis (UNC-CONICET), Cecilia Herrera (UNC) and Jorge Lauret (UNC-CONICET).
Junior: Alejandro Tolcachier (PhD, UNC-CONICET).

Brazilian Team

Senior: Henrique Sá Earp, (UNICAMP), Lino Grama (UNICAMP), Marcos Jardim (UNICAMP) Charles A. de Almeida (UFMG), Maurício B. Corrêa (UFMG), Renato Vidal Martins (UFMG), Aline V. de Andrade (UFF), Gonzalo de Oliveira (UFF), Simon Chiossi (UFF), Andrew Clarke (UFRJ), Lázaro Rodriguez-Diaz (UFRJ), Abdel M. A. Henni (UFSC),
Junior: Andrés Moreno (Post-doc, UNICAMP), Daniel Fadel (Post-doc, UFF), Brian Grajales (PhD, UNICAMP), Paola Saavedra (PhD, UNICAMP), Luis Portilla (PhD, UNICAMP).

Chilean Team

Senior: Elizabeth Terezinha Gasparim (UCN)
Junior: Bruno Suzuki (Post-doc,UCN-USP) and Francisco Iván Rubilar Arriagada (PhD-UCN).

French Team

Senior: Viviana del Barco (UPSaclay), Eric Loubeau (UBO, Brest), Andrei Moroianu (CNRS/UPSaclay), Jean Vallès (UPPA, Pau), Daniele Faenzi (UB, Dijon), Eveline Legendre (IMT, Toulouse).
Junior: Grégoire Menet (Post-doc, IF, Grenoble), Hiba Bibi (PhD, UBO), Brice Flamencourt (PhD, UPSaclay).

A9	International Project Coordinator (to be chosen among the Scientific Coordinators mentioned in A6)
	Viviana del Barco, Université Paris Saclay, France, and CONICET, Argentina.

B. Project Details

B1. Project guidelines

We propose a collaboration of four countries to advance the understanding of special geometric structures in differentiable manifolds and to contribute to possible resolutions of three classic problems in algebraic geometry that have moduli spaces as background.

On the side of differential geometry, the project will focus on the study of (locally conformally) Kähler and G_2 structures and their close links to higher dimensional gauge theory. Contributions in the area of algebraic geometry will point towards the study of foliations and holomorphic distributions in projective spaces, moduli spaces in abelian varieties and hyperplane arrangements in projective spaces. The project aims to develop these topics, contributing to the general development in Geometry & Topology, in the lines of research that are considered main trends. This is emphasized by none other than Sir Simon Donaldson, in his recent plenary speech at ICM2018, in Rio de Janeiro, who, in particular, mentioned both G_2 geometry and extremal Kähler metrics.

B2. Project description

1 Goals, motivation, methodology and contribution of each participating institution

1.1 Harmonic geometric structures - General theory

A recent collaboration between the project's members Sá Earp and Loubeau [LSE19] formulated a general theory of harmonicity for geometric structures on a Riemannian manifold (M^n, g), with structure group $G \subset \mathrm{SO}(n)$, building upon a framework originally outlined in [Woo97, Woo03], and further considered eg. in [GDMC09], along with the first steps towards an analytic theory of their associated *harmonic section flow* (HSF). From the outset, that theory was able to unify a significant number of results scattered across several pieces of recent literature eg. [Bag19, Gri17, Gri19, DGK19, He19, HL19]. This opens up a number of paths, on one hand, towards the further understanding of the flow itself, and on the other, regarding a number of interesting particular contexts, in which various torsion regimes, and eg. their relation to integrability, can be better understood.

The article [LSE19] also proposes a novel approach to the classical and important question in Riemannian geometry of defining the concept and find 'the best' geometric structure in a given class. Their appoach exploits the correspondence of geometric structures on differentiable manifolds with *sections* of a certain homogeneous fibre bundle, correspondence which emerges by the reduction of the structure group. A natural Dirichlet energy functional can be assigned to these sections by means of the *vertical tension* of a fibre bundle.

A geometric structure is defined to be *harmonic* if the corresponding section is a critical point of the energy functional, and *torsion-free* if the vertical tension vanishes. These conditions on the vertical tension can be expressed through a natural geometric PDE and is typically a weaker condition than the corresponding notion of 'integrability' of the geometric structure. Thus in favourable cases harmonicity can still characterise the 'best' geometric structure in a given class, even when 'integrability' is otherwise obstructed or trivial.

Loubeau and Sá Earp showed that the HSF is closely related to the *harmonic map heat flow* (HMHF) of equivariant maps on sections of the structure bundle, for which uniqueness and short-time existence of solutions and behavioural results (like short-time blow-up) of the flow were established in [CD90].

The unification of a significant number of results through the analytic theory of the *harmonic section flow* opens up a number of paths, on one hand, towards the further understanding of the flow itself, and on the other, regarding a number of interesting particular contexts, in which various torsion regimes, and eg. their relation to integrability, can be better understood.

For instance, generalized complex geometry was introduced by Hitchin [Hit03] and later developed by Gualtieri [Gua03] as a unifying theory for almost-complex and almost-symplectic geometry, thus achieving a description of both structures as linear operations in the so-called bitangent bundle. In our perspective, it is given by a $\mathrm{O}(2n, 2n)$-structure $H = \mathrm{U}(n, n)$, and it would be interesting to identify the harmonicity condition detecting the 'best' generalised structure for a fixed metric on the bitangent bundle.

An important set of questions regarding long-term existence and convergence for HMHF remain open, namely whether a small 'entropy' condition suffices to ensure eventually regular bounded torsion and hence convergence to a harmonic (or even torsion-free) limit. Such results for HSF were established in [CD90].

Results in this direction are of particular relevance in view of what is known, for instance, in the case of G_2-structures (see below). Such progress could not be obtained right away because, even if the initial section for the HSF has 'small energy' in a seemingly suitable sense, the initial energy of the associated section of the HMHF is practically never small. This indicates that the equivariance of this section must play a much more central role in the analysis, a theory which cannot be as of yet found in the literature. In this context, we aim to explore possible 'small energy' conditions for equivariant sections which guarantee long term existence and regularity of solutions of HMHF.

As another instance, the novel approach proposed by the team member Rodríguez-Díaz to study the existence problem of integrable almost complex structures on $\mathbb{S}^6$ via Kirchhoff's theorem [Día] can be addressed as a problem of characterisation of torsion for $\{e\}$-structures on spheres. This approach takes into account the various torsion regimes for a geometric section: from the annihilation of the vertical tension up to harmonic sections a whole chain of cases. We know from examples, that integrability fits somewhere strictly within that chain. On the other hand, in each specific context some of these conditions collapse into equivalence. Therefore it makes sense to ask, somewhat philosophically, where does integrability fit in this chain. Equivalently, in each context, what is the obstruction to integrability of a G-structure ?

In our context of particular interest, when G is trivial, an $\{e\}$-structure (i.e. a *parallelism*) on (M, g) is equivalent to a global section $\sigma : M \to P_{\mathrm{SO}(n,g)}$. Such parallelism is *integrable* if, and only if, the torsion of the associated zero curvature connection ∇^c is parallel with respect to ∇^c, in which case M is a Lie group. In particular, for parallelisms on a sphere $\sigma : (\mathbb{S}^n, g) \to \mathrm{SO}(n+1, g)$, it is not hard to check that eg. the Hopf frame on round $\mathbb{S}^3$ is harmonic as a section and integrable, but $d^{\mathcal{V}}\sigma \neq 0$, since it is non-Abelian.

Although this new understanding of torsion for frame fields would be interesting and publishable on its own, there is another (very optimistic) possible outcome: once the full analytic theory for our flow is available, one might consider flowing from the Kirchhof frame field on the 7-sphere induced by a conjectural complex structure on $\mathbb{S}^6$, hopefully converging to an integrable parallelism, thus proving by contradiction Hopf's famous conjecture.

Expected results	1. *Monotonicity and regularity for G-equivariant maps under the harmonic map heat flow of a G-equivariant maps.* 2. *Establish a harmonicity condition for generalised complex structures.* 3. *Compare torsion conditions and integrability for parallelisms on a sphere $(\mathbb{S}^n, g)$; Relation between integrability of complex structures on $\mathbb{S}^6$ and associated parallelism on $\mathbb{S}^7$.*

1.2 Homogeneous G_2-structures and G_2-flow

The study of the harmonicity condition of general geometric structures has been inspired by an instance which has recently attracted significant attention, in the realm of G_2-geometry. Its objects of highest interest are G_2-manifolds, i.e. Riemannian 7-manifolds (M, g_φ) with holonomy in the exceptional Lie group G_2 [Joy96]. This reduction amounts to solving a difficult non-linear PDE, but it may be relaxed in a number of ways by considering the fundamental notion of a G_2-*structure*: a non-degenerate 3-form φ which induces a so-called G_2-*metric* g_φ. The failure of this metric to give rise to a metric with holonomy G_2 is measured by the geometric object known as the full torsion tensor T: vanishing torsion is equivalent for φ to be simultaneously closed and coclosed. Several authors focused their study in relaxations of the G_2 holonomy condition, treating the study of closed or coclosed non-degenerate 3-forms on 7-dimensional manifolds.

The seminal article by Grigorian [Gri17] argues that the divergence-free torsion condition (Div $T = 0$) should be interpreted as a 'gauge-fixing' among isometric G_2-structures, hence G_2-structures with divergence-free torsion are naturally 'the best' among all those inducing the same Riemannian metric. Loubeau and Sá Earp offer an alternative formulation of the divergence-free torsion equation as a harmonicity condition on sections of a natural projective-bundle where the harmonic section flow (HSF) corresponds to the so-called isometric flow of G_2-structures or Div T-flow already present in the theory [DGK19]. A good understanding of long-time existence of the Div T-flow, at least in a large number of examples, would significantly advance us towards fixing the parabolicity of the so-called Laplacian coflow, which could optimistically converge to torsion-free structures starting from coclosed ones, thereby producing new solutions to this difficult problem.

The homogeneous setting $M^7 = G/H$ is a way to mass-produce examples of harmonic G_2-structures with good HHG$_2$F features. Relying on classification results, we aim to determine if the divergence-free torsion condition is more general than the nearly-G_2 condition for coclosed G_2-structures. Symmetries of the space reduce the geometric flow to an ODE [Lau16]. Working in this context could lead to an interesting classification programme based on the Gromov-Hausdorff limits introduced recently by Lauret [Lau12, Lau16], especially in cases for which the existence of (non-flat) geometric structures with G_2 holonomy is known to be obstructed. We aim to consider, in particular, homogeneous seven dimensional manifolds and study the HSF from the perspective introduced by Lauret in the general context of geometric flows of invariant tensors on homogeneous spaces. Then a guiding philosophical question is: *Under which conditions can harmonic, or equivalently divergence free-torsion,* G_2*-structures exist on a homogeneous 7-manifold?*

Obstructions are known for some particular cases. For instance, closed harmonic G_2-structures cannot occur on compact homogeneous manifolds. To the contrary, nearly-G_2-structures, whose defining 3-form is Killing and which are also harmonic, can only occur on compact manifolds, if assumed homogeneous. Homogeneous coclosed compact G_2-manifolds were classified by Reidegeld [Rei10], who also recovers the previous classification of homogeneous nearly-G_2-structures from [FKMS97]. Further homogeneous G_2-structures arise from left-invariant positive 3-forms on solvable Lie groups. Among them, closed and coclosed structures have been studied by several authors. In contrast to the compact case, left-invariant closed G_2-structures exist and have been classified on special classes of solvable Lie groups. In searching for harmonic coclosed G_2-structures, in which case the torsion T is symmetric, one can look for situations in which T is a symmetric Killing tensor. Since every symmetric Killing 2-tensor with constant trace is divergence-free, this would provide more examples of harmonic G_2-structures. Left-invariant symmetric Killing 2-tensors on nilpotent Lie groups where studied by del Barco and Moroianu [BM].

When working in the homogeneous case, it is natural to impose the homogeneity to the solutions of the Div T-flow. Under this assumption, the flow of isometric G_2-structures becomes an ODE and the evolution equation is called *homogeneous harmonic* G_2*-flow* (HHG_2F). After identifying harmonic coclosed G_2-structures on homogeneous compact manifolds within Reidegeld's classification, we aim to consider the natural problem of classifying those ones which, in addition, carry a solution of the HHG_2F. On the other hand, the Lie bracket flow introduced by Lauret in the recent years is a dynamical system which can be defined on the variety of Lie algebras, corresponding to a large class of geometric flows. It is our purpose to compute the bracket flow associated to the Div T-flow of coclosed G_2-structures. Moreover, we pursue to study the limit of the HHG_2Fon compact homogeneous manifolds carrying non-harmonic coclosed G_2-structures. In this context, team members Moreno and Sá Earp used a natural ansatz of Karigiannis et al. [KMT12] to construct a self-similar solution, or *soliton*, of the Laplacian co-flow on a solvable Lie group of the form $M^7 = N^6 \times L^1$, where $L^1 = \mathbb{R}$ or $\mathbb{S}^1$ and N^6 is compact and either nearly-Kähler or a Calabi-Yau 3-fold [MSE19]. Under the flow, such G_2-structures simply scale monotonically and move by diffeomorphisms, yet they provide potential models for singularities of the flow, as well as means for desingularising certain singular G_2-structures, both key aspects of any geometric flow.

Expected results	1. *Examples of divergence free-torsion* G_2*-structures on homogeneous 7-manifolds.* 2. *Classify harmonic coclosed* G_2*-structures within Reidegeld's lists and study the solutions of the* HHG_2F. 3. *Classify coclosed harmonic* G_2*-structures on solvable Lie groups.* *Identify the properties of the limit structure of the bracket flow associated to the Div* T*-flow of coclosed* G_2*-structures.* 4. *Study the Killing condition on the total torsion* T *of harmonic coclosed* G_2*-structures on solvmanifolds.*

1.3 Locally conformally Kähler manifolds

The most important class of Hermitian manifolds is definitely the class of Kähler manifolds. Given a compact smooth manifold, there are topological obstructions to the existence of a Kähler metric and it is known that many complex manifolds cannot admit Kähler metrics. A larger class of Hermitian manifolds is given by the locally conformal Kähler (LCK) manifolds, which have shown to be of great importance lately. These are Hermitian manifolds such that each point has a neighbourhood where the metric is conformal to a Kähler metric. Equivalently, a Hermitian manifold (M, J, g) is LCK if and only if there exists a closed 1-form θ such that $d\omega = \theta \wedge \omega$ where ω is the fundamental 2-form. In this case, the 1-form θ is called the Lee form. A Hermitian manifold (M, J, g) is called globally conformally Kähler (GCK) if there exists a C^∞ function, $f : M \to \mathbb{R}$, such that the metric $\exp(-f)g$ is Kähler, or equivalently, the Lee form is exact. Therefore a simply connected LCK manifold is GCK.

Some other facts about LCK manifolds are: (i) they belong to the class $\mathcal{W}_4$ of the Gray-Hervella classification of almost Hermitian manifolds [GH80]; (ii) in contrast to the Kähler class, the LCK class is not stable under small deformations [Bel00], but, just as in the Kähler case, the LCK class is stable under blowing-up points (see [Tri82, Vul09]).

The classical Hopf manifolds are examples of LCK manifolds, and they are obtained as a quotient of $\mathbb{C}^n - \{0\}$ with the Boothby metric by a discrete subgroup of automorphisms. These manifolds are diffeomorphic to $S^1 \times S^{2n-1}$ and for $n \geq 2$ they do not admit any Kähler metric. The LCK structures on these Hopf manifolds have a special property, as shown by Vaisman in [Vai79]. Indeed, the Lee form is parallel with respect to the Levi-Civita connection of the Hermitian metric. The LCK manifolds sharing this property form a distinguished class, which has been much studied since Vaisman's seminal work [Bel00, GMO15, KS80, OV03, Vai79, Vai82]. Indeed, LCK manifolds with parallel Lee form are nowadays called *Vaisman manifolds*. Vaisman manifolds satisfy stronger topological properties than general LCK manifolds. For instance, a compact Vaisman non-Kähler manifold (M, J, g) has $b_1(M)$ odd ([KS80, Vai82]), which implies that such a manifold cannot admit Kähler metrics.

A well known family of non-Kähler compact complex manifolds is given by the Oeljeklaus-Toma manifolds (OT for short), introduced in [OT05]. These manifolds arise from certain number fields, and they can be considered as generalizations of the Inoue sufaces of type S^0. Some of them admit LCK metrics, and among them there are counterexamples to a conjecture formulated by I. Vaisman, which stated that any compact LCK manifold has odd first Betti number. These manifolds have many interesting properties; for instance, it was proved in [OT05] that the following holomorphic bundles over an OT manifold X are flat and admit no global holomorphic section: the bundle of holomorphic 1-forms Ω^1_X, the holomorphic tangent bundle $\mathcal{T}_X$ and any positive power $K_X^{\otimes k}$ of the canonical bundle. Moreover, the Kodaira dimension of any OT manifold is $-\infty$ and they can never be Kähler. Later, it was proved in [Kas13] that they cannot admit any Vaisman metric.

A natural family of compact manifolds where to study the LCK condition is given by the nilmanifolds and solvmanifolds. In the survey article [AO19] there is a compilation of known results about this topic. We review some of them here. We recall that a discrete subgroup Γ of a simply connected Lie group G is called a *lattice* if the quotient $\Gamma\backslash G$ is compact. According to [Mil76], if such a lattice exists then the Lie group must be unimodular. The quotient $\Gamma\backslash G$ is known as a solvmanifold if G is solvable and as a nilmanifold if G is nilpotent, and it is known that $\pi_1(\Gamma\backslash G) \cong \Gamma$ (hence they are not simply connected). Moreover, the diffeomorphism class of solvmanifolds is determined by the isomorphism class of the corresponding lattices.

There are many interesting results about the Hermitian geometry of nilmanifolds and solvmanifolds, especially when the Hermitian structure is induced by a left invariant one on the corresponding Lie group. For instance, it was proved in [BG88] that a non-toral nilmanifold cannot admit Kähler structures. This was later generalized to completely solvable solvmanifolds by Hasegawa in [Has06], where he also proves that Kähler solvmanifolds are finite quotients of complex tori which have the structure of a complex torus bundle over a torus. Also, the first example of a compact symplectic manifold that does not admit any Kähler metric happens to be a nilmanifold, the well known Kodaira-Thurston manifold, which is a primary Kodaira surface ([Kod64, Thu76]).

Concerning LCK (or Vaisman) structures on nilmanifolds/solvmanifolds, we have:

- the only nilmanifolds with an invariant LCK structure are quotients of $H_{2n+1} \times \mathbb{R}$, where H_{2n+1} is the $(2n+1)$-dimensional Heisenberg group [Saw07]. Moreover, they are Vaisman;
- the only nilmanifods with a (non-necessarily invariant) Vaisman structure are the same as in the previous item [Baz17];
- 4-dimensional solvmanifolds with invariant LCK structures were determined in [HK15];
- OT manifolds are in fact solvmanifolds, where the Hermitian structure is actually invariant;
- LCK solvmanifolds with abelian complex structure are also quotients of $H_{2n+1} \times \mathbb{R}$, and they are Vaisman [AO15];
- LCK solvmanifolds where the Lie group is almost abelian (i.e. its Lie algebra has a codimension one abelian ideal) occur only in dimension 4 [AO18];
- solvmanifolds with invariant Vaisman structures were studied in [AO17] ; a concrete description of their Lie algebras in terms of Kähler flat Lie algebras was given. A classification in dimension 6 was also attained.

The goals of this project concerning LCK solvmanifolds are twofold. On the one hand we pursue to find new examples of invariant LCK structures on solvmanifolds. This has two steps: first, to determine solvable

Lie algebras which admit LCK structures. We will first develop a constriction method to produce LCK Lie algebras beginning with another one of lower dimension. The second step is to determine if the simply connected Lie groups associated these Lie algebras admit a lattice. In general, it is not easy to establish whether a given solvable Lie group admits a lattice or not. To answer this question, we will need tools of Lie theory and algebraic number theory.

On the other hand we aim to determine general geometric properties of LCK solvmanifolds which do not admit Vaisman metrics. OT manifolds with LCK metrics are in this family, and it would be very interesting to find out if the other solvmanifolds share the same properties. In particular: to find whether the bundle of holomorphic 1-forms, the holomorphic tangent bundle and any positive power of the canonical bundle are trivial or not. Also, to establish if the Kodaira dimension of such a solvmanifold is $-\infty$ or not. Finally, it would be interesting to know if non-Vaisman LCK solvmanifolds admit non-trivial homolorphic vector fields. It is known that OT manifolds do not admit such a vector field; note that LCK structures with holomorphic Lee vector field on Vaisman-type manifolds were studied in [MMP19].

Expected results	1. *New methods to construct unimodular Lie algebras with LCK structures.* 2. *Existence of lattices in some of the corresponding Lie groups.* 3. *Triviality of some holomorphic vector bundles on LCK solvmanifolds.* 4. *Holomorphic vector fields on non-Vaisman LCK solvmanifolds.*

1.4 Gauge theory on Kähler and G_2-geometries

Tian's paper [Tia00] introduces a general anti-self-duality equation, for connections A on a G-bundle $E \to M$ over an oriented Riemannian manifold (M^n, g) endowed with a closed $(n-4)$-form Ξ. A connection is called a Ξ-*instanton* if its curvature satisfies $*(\Xi \wedge F_A) = -F_A$. This generalises the ASD equations in 4-dimensions, the HYM equations on Kähler manifolds, among others. Tian thus related gauge theory to calibrated geometry (cf. [HL82]): when Ξ is a calibration, the blow-up set of a sequence of Ξ-instantons defines a Ξ-calibrated submanifold [Tia00, Theorem 4.3.2]. In 7-dimensional G_2-geometry, the 3-form $\Xi = \varphi$ defines the G_2-*instanton equation*. A meaningful compactification of the moduli space of G_2-instantons would lead to enumerative invariants through an appropriate 'counting' of G_2-instantons and *associative* submanifolds — which are φ-calibrated [Wal13] (see also [MSE]). This 'instanton count' should resemble the Casson invariant and instanton Floer homology for flat connections on 3-manifolds [DK90, DT98]. However, major compactification issues remain, and it remains relevant to construct a large amount of working examples.

An important method to produce compact manifolds with $\mathrm{Hol}(g) = \mathrm{G}_2$ is the *twisted connected sum construction* (TCS), suggested by Donaldson and developed by [Kov03, KL11, CHNP13, CHNP15]. A *building block* consists of a projective 3-fold Z and a smooth anti-canonical $K3$ surface $\Sigma \subset Z$ with trivial normal bundle. Given a hyperKähler structure $(\omega_I, \omega_J, \omega_K)$ on Σ such that $[\omega_I]$ is the restriction of a Kähler class on Z, one can make $V := Z \setminus \Sigma$ into an asymptotically cylindrical (ACyl) Calabi-Yau 3-fold, with a tubular end modelled on $\mathbb{R}_+ \times \mathbb{S}^1 \times \Sigma$ [HHN15]. Then $Y := \mathbb{S}^1 \times V$ is an ACyl G_2-manifold with a tubular end modelled on $\mathbb{R}_+ \times \mathbb{T}^2 \times \Sigma$. When a pair of building blocks *matches* 'at infinity', one can glue $Y_\pm$ by interchanging the $\mathbb{S}^1$-factors. This yields a simply-connected *closed* manifold with a family of torsion-free G_2-structures $(Y^7, \phi_T)_{T \geq T_0}$, having a 'long neck' modelled on $[-T, T] \times \mathbb{T}^2 \times \Sigma_+)$. This construction raises a programme in gauge theory, aimed at constructing G_2-instantons over compact TCS, originally outlined in [SE09].

If $\mathcal{E} \to (Z, \Sigma)$ is a reflexive sheaf over a building block, such that $\mathcal{E}|_\Sigma$ is a stable holomorphic bundle, then $\mathcal{E}|_\Sigma$ carries a unique compatible ASD instanton [Don85], away from the singular locus of $\mathcal{E}$. In this situation, $\mathcal{E}|_V$ can be given a HYM connection asymptotic to the ASD instanton on $\mathcal{E}|_\Sigma$ [SE15, Theorem 58] & [JW18, Theorem 1.1], whose pullback to $\mathbb{S}^1 \times V$ is a G_2-instanton. It is possible to glue a suitable pair of such *smooth* solutions into a G_2-instanton over the *compact* twisted connected sum, provided a number of rather technical conditions are met [SEW15, Theorem 1.2], the most constraining of which concerns transversality of the bundles over the $K3$ surface Σ at infinity. However, in [MNSE], we use the so-called Hartshorne-Serre construction to obtain glueable families of bundles over the building blocks, for which the gluing is *nontrivially transversal*. A natural development would be to adapt the gluing theorem [SEW15, Theorem 1.2] to pairs of singular Hermitian-Yang-Mills connections as in [JW18, Theorem 1.1], by a careful understanding of the deformation theory of reflexive sheaves. In addition, as a sequel to [MNSE], we aim to produce new pairs of glueable asymptotically stable reflexive sheaves, by extending the Hartshorne-Serre construction.

In a different direction, we will explore generalizations of the gluing construction, using other types of building blocks, such as toric local Calabi–Yau 3-folds. The first example will be the resolved conifold $\mathrm{Tot}(\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(-1))$. In further generality we can consider the local Calabi–Yau 3-folds $W_k = \mathrm{Tot}(\mathcal{O}_{\mathbb{P}^1}(-k) \oplus \mathcal{O}_{\mathbb{P}^1}(k-2))$ which have infinite dimensional families of deformations, containing infinitely many distinct isomorphism types, see [Suz19, GKRS18]. The 3-folds belonging to these families can be glued by holomorphic procedures, and thus may be regarded as new building blocks, which when promoted to a 7-manifold, say after product with the circle, might generate interesting new manifolds with G_2-metrics. Note that a crepant resolution of a Calabi–Yau 3-fold X can be obtained by holomorphic gluing of such local models W_k, therefore this generalisation also allows us to study the effect of replacing the 3-fold by one of its birationally equivalent models. It is worth mentioning that the Chilean team has studied in detail vector bundles on these local 3-folds, their deformation theory and their moduli spaces [BGS19, GR19], and promoting this study to vector bundles on manifolds with G_2-metrics is likely to be a very fruitful endeavour.

Drawing more fundamentally from the potential applications in Theoretical Physics, G_2-geometry relates directly to problems from M-theory and heterotic string theory. For example, the construction of '$\mathcal{N} = 1/2$ BPS domain walls' by solving the so-called G_2-Strominger system of nonlinear PDEs involving the G_2-structure and a connection in an auxiliary bundle satisfying a 7-dimensional analogue of the ASD equations.

Expected results	1. *Sufficient gluing conditions for singular* G_2*-instantons over TCS*; *Mass-producing examples of matching pairs of reflexive sheaves.* 2. *New families of G_2 geometries fibered over Calabi–Yau threefolds obtained via holomorphic gluing.* 3. *Solving the* G_2*-Strominger system on* 3*-Sasakian mfds or by reduction to ODE.* 4. *Classifying solutions of the* G_2*-Strominger system on Aloff-Wallach mfds.*

1.5 Kähler reductions, canonical metrics and instantons

There is a well-established bridge between G_2-holonomy metrics and Kähler 6-manifolds. When a G_2-manifold (Y^7, g_ϕ) admits a circle action $\mathbb{S}^1 \curvearrowright Y$ by isometries, one can form a Kähler variety as the quotient $Y/\mathbb{S}^1$, e.g. [AS04]. This suggests that many features of Kähler manifolds should admit counterparts in G_2-geometry, as suggested by Donaldson's plenary address to the ICM2018. We will investigate further dimensional reduction techniques in the context of gauge theory and canonical Kähler metrics.

A *Kähler-Einstein (K-E) metric* on a Kähler manifold (X, ω) is a solution to $\mathrm{Ric}(\omega) = c\omega$, where $\mathrm{Ric}(\omega)$ is the Ricci form, $c \in \mathbb{R}$. K-E metrics satisfy Einstein's equation, and in that respect can be considered the Kähler analogue of G_2-metrics. By fundamental results of Yau and Aubin, solutions for $c = 0$ (resp. $c < 0$) are known to exist, provided $c_1(X)$ vanishes (resp. is negative). For Fano varieties (with $c_1(X) > 0$), the recent solution of the Yau-Tian-Donaldson conjecture [Yau93, Tia97, Don02] shows that K-stability is equivalent to the existence of a solution, as an analogue for varieties of the Kobayashi-Hitchin correspondence for bundles. Unfortunately, it can be very difficult to understand whether a Fano variety is K-stable, and it remains crucial to understand the behaviour of K-E varieties under complex transformations of the polarized manifold, eg. their stability under complex deformations can be done by perturbation techniques [RT14, CT14]. It would be interesting to know whether K-E metrics survive under more drastic changes of the manifold. As many Fano manifolds carry continuous symmetries, it is natural to consider Kähler reductions. This was first studied by Futaki in [Fut87], but without the recent K-stability technology. With this new tool at hand, Clarke and Sá Earp, from the Brazilian side, together with Legendre, from the French part, will revisit the problem. As some technical tools shall be developed along the way, we will focus first on the easier case of Hermite-Einstein metrics on vector bundles, that are instantons in this Kähler context.

Geometric Invariant Theory (GIT) provides an important toolkit to take quotients in algebraic geometry [MFK94]. Suppose a reductive algebraic group G acts on a complex projective variety $X \subseteq \mathbb{CP}^n$ by restriction of a linear action of $G \subseteq \mathrm{SL}(n+1, \mathbb{C})$ on $\mathbb{CP}^n$. Suppose that G acts on the polarized variety (X, H), and that $\mathcal{E} \to X$ is a G-equivariant holomorphic vector bundle. According to known criteria under which $\mathcal{E}$ descends to the algebraic quotient [DN89, Nev08], we suppose that $\mathcal{E}|_{X^{ss}} = \pi^* \mathcal{E}_Y$, for a bundle $\mathcal{E}_Y$ on $Y = X//G$. It is natural to expect a relation between stability of $\mathcal{E}$ over (X, H), stability of $\mathcal{E}_Y$, and geometric properties of the representation $G \to \mathrm{Aut}(X, L)$. Indeed, in [CT], Clarke (**Br team**) and Tipler (**Fr team**) obtained such a relation in the equivariant context of toric varieties. Furthermore, they provide a fully faithful embedding of the category of torus-equivariant stable bundles on Y to the category of torus-equivariant stable bundles on X that descend to Y, provided that the G-action satisfies a *Minkowski condition*. Similar results of this type are known in other contexts, eg. the existence of certain gauge fields is preserved under reduction [GP94], and a subcategory of G-invariant stable bundles on one space is equivalent to a subcategory of stable parabolic bundles on its reduced space [Mun02].

A natural generalization of [CT] to non-toric varieties would require the appropriate geometric condition on $G \to \mathrm{Aut}(X, H)$ for the category of stable torsion-free sheaves on Y to embed into the category of G-equivariant stable sheaves on X. Moreover, one would like to extend this correspondence to circle reductions $Y \to X = Y/\mathbb{S}^1$ where (Y, g_ϕ) carries an $\mathbb{S}^1$-invariant G_2-metric g_ϕ and X is a Kähler manifold. A spin-off initiative can henceforth be foreseen, in synergy with Work Package 4, in which case Sá Earp and Menet would a posteriori join this team, we shall investigate a possible adaptation of the Hartshorne-Serre construction of slope-stable vector bundles to the context of GIT quotients to lift examples from Y to X.

Going back to varieties and K-E metrics, it is natural to expect a similar relation between the K-stability of a Fano variety X, of its GIT quotient $Y = X//G$ (which is Fano from Futaki's work [Fut87]), and of the algebro-geometric properties of the representation $G \to \mathrm{Aut}(X)$. The expertise of Legendre in toric geometry will be valuable in understanding an adaptation Minkowski condition of [CT] be adapted to the context of Kähler reduction of toric K-E metrics. Of course, once this question settled in the toric context, it is natural to extend it to non-toric varieties.

Expected results	1. *Embedding of stable torsion-free sheaves into G-equiv. stable sheaves; Reduction and Hartshorne-Serre construction.* 2. *Minkowski condition for quotients of* G_2*-manifolds.* 3. *Minkowski condition for K-E reduction.*

1.6 Moduli spaces in algebraic geometry and applications

We will work on three classical problems in algebraic geometry which have moduli spaces as background: the study of holomorphic foliations and distributions in projective spaces, moduli spaces of sheaves, and logarithmic sheaves on plane and space curves.

1.6.1 Holomorphic foliations and distributions in projective varieties

Recall that a *codimension r distribution* $\mathcal{F}$ on a nonsingular projective variety X is given by an exact sequence

$$\mathcal{F}:\ 0 \longrightarrow T_{\mathcal{F}} \xrightarrow{\phi} TX \xrightarrow{\pi} N_{\mathcal{F}} \longrightarrow 0, \tag{1}$$

where $T_{\mathcal{F}}$ is a coherent sheaf of rank $s := \dim(X) - r$, and $N_{\mathcal{F}}$ is a torsion free sheaf. The sheaves $T_{\mathcal{F}}$ and $N_{\mathcal{F}}$ are called the *tangent* and the *normal* sheaves of $\mathcal{F}$, respectively. Note that $T_{\mathcal{F}}$ must be reflexive.

The *singular scheme* of $\mathscr{F}$ is defined as follows. Taking the maximal exterior power of the dual morphism $\phi^\vee \colon {\Omega^1}_X \to {T_{\mathcal{F}}}^\vee$ we obtain a morphism $\Omega^s_X \to \det(T_{\mathcal{F}})^\vee$; the image of such morphism is the ideal sheaf $I_{Z/X}$ of a subscheme $Z \subset X$, the singular scheme of $\mathscr{F}$, twisted by $\det(T_{\mathcal{F}})^\vee$.

Furthermore, a *foliation* is an integrable distribution, that is a distribution

$$\mathscr{F}:\ 0 \longrightarrow T_{\mathcal{F}} \xrightarrow{\phi} TX \xrightarrow{\pi} N_{\mathcal{F}} \longrightarrow 0$$

whose tangent sheaf is closed under the Lie bracket of vector fields, i. e. $[\phi(T_{\mathcal{F}}), \phi(T_{\mathcal{F}})] \subset \phi(T_{\mathcal{F}})$. Clearly, every distribution of codimension $\dim(X) - 1$ is integrable.

Techniques from algebraic geometry have been widely used by several authors in the study of holomorphic distributions and foliations in projective varieties, like the study of the singularities of foliations, and the geometry of the tangent and conormal sheaves.

The main goal of this sub-project is to obtain a classification of distributions and foliations of low degree on projective varieties of dimension 3, especially $\mathbb{P}^3$ and hypersurfaces in $\mathbb{P}^4$, and higher-dimensional homogeneous varieties, like grassmannians. In [CACJ20], the authors provided a full classification of codimension one distributions of degree 1 on $\mathbb{P}^3$, and a partial classification of codimension one distributions of degree 2 on $\mathbb{P}^3$. We now hope to complete the classification of codimension one distributions of degree 2 on $\mathbb{P}^3$, extending the celebrated classification of codimension one foliations of degree 2 on $\mathbb{P}^3$ by Cervau and Lins Neto [CL96], and start a classification of codimension one distributions of degree 3 on $\mathbb{P}^3$.

Our research group has also worked on codimension two foliations on $\mathbb{P}^3$ (called *foliations by curves*), providing a characterization of those foliations of degree ≥ 3 whose conormal sheaf is locally free [CJM]. Next, we expect to achieve a full classification of foliations by curves of degree 1 and 2.

We also expect to address two of the main open questions regarding codimension one foliations on the 3-dimensional projective space, namely: 1) the 1-dimensional component of the singular scheme of a codimension one foliation is connected, and the first cohomology module of the tangent sheaf is trivial; if the tangent sheaf of a codimension one foliation is locally free, then it splits as a sum of line bundles.

Expected results	1. *Classification of codimension one distributions of degree 2 on* $\mathbb{P}^3$ 2. *Classification of foliation by curves of degree 2* 3. *Classification of codimension one distributions of degree 0 on grassmannians*

1.6.2 Moduli spaces and deformations of sheaves in projective varieties.

Moduli spaces of sheaves on projective varieties have long been one of the central topics of research in algebraic geometry. However, many open problems remain, especially for varieties of dimension 3 and larger. We plan to focus on the connectedness of the moduli space $\mathcal{M}(c_1, c_2, c_3)$ of rank 2 sheaves with fixed Chern classes (c_1, c_2, c_3) on the 3-dimensional projective space.

In general, $\mathcal{M}(c_1, c_2, c_3)$ has many irreducible components, and it is known for certain, very specific values of the Chern classes that $\mathcal{M}(c_1, c_2, c_3)$ is connected. By contrast, the Hilbert scheme with arbitrary Hilbert polynomial, which coincides with the moduli space of rank 1 sheaves with any Chern classes, is always connected according to a result by Hartshorne from the late 1960's.

Underlying the issue of connectedness is the need for an understanding how stable locally free or reflexive sheaves degenerate into torsion free sheaves, and conversely when a stable sheaf can be smoothed into a locally free or reflexive sheaf. These sheaves, called *smoothable sheaves*, are the ones lying in the intersection of two irreducible components of $\mathcal{M}(c_1, c_2, c_3)$, so one way to approach the connectedness problem is to characterize smoothable sheaves and try to argue that any stable rank 2 sheaf can be deformed into a smoothable sheaf. Another approach is to related (subsets) of the moduli space $\mathcal{M}(c_1, c_2, c_3)$ to (subsets) of the Hilbert scheme of 1-dimensional schemes, which is known to be connected, via the well-known Serre correspondence. One case in which the these approaches can be tested is $\mathcal{M}(0, n, n^2 - n)$ for each $n \geq 2$, which is expected to have only two irreducible components intersecting one another.

A more limited goal is to study the connectedness of the moduli space of instanton sheaves, which is an open subset of $\mathcal{M}(0, n, 0)$. This is amenable to other techniques, like the study of fixed point set for a torus action as pursued by Henni [Hen], or a more traditional, sheaf-theoretic approaches used by Jardim-Maican-Tikhomirov [JMT17].

Expected results	1. *Characterization of connected clusters of irreducible components of* $\mathcal{M}(0, n, 0)$ *linking different components of locally free sheaves.* 2. *Proof of connectedness of* $\mathcal{M}(0, n, n^2 - n)$.

1.6.3 Logarithmic sheaf and special plane curves.

Let S and T be the surfaces in $\mathbf{P}^3$ respectively defined by the equations $\{f = 0\}$ and $\{g = 0\}$, of degree $s + 1$ and $t + 1$, respectively, with $0 \leq s \leq t$. We then consider the Jacobian matrix:

$$\mathcal{J}_{f,g} := \begin{bmatrix} \partial_0 f & \partial_1 f & \partial_2 f & \partial_3 f \\ \partial_0 g & \partial_1 g & \partial_2 g & \partial_3 g \end{bmatrix} : \mathcal{O}_{\mathbb{P}^3}^{\oplus 4} \to \mathcal{O}_{\mathbb{P}^3}(s) \oplus \mathcal{O}_{\mathbb{P}^3}(t) \tag{2}$$

Under certain conditions (like $s = 0$ or $s = t$), the sheaf $\ker \mathcal{J}_{f,g}$ only depends on the ideal generated by the polynomials f and g, and can be therefore regarded as intrinsic to the complete intersection curve; in this situation, we use the notation $\mathcal{T}_C := \ker(\mathcal{J}_C)$. The sheaf $\mathcal{T}_C$ is called, by analogy with the logarithmic sheaf associated to a hypersurface in $\mathbb{P}^n$, the *logarithmic sheaf* associated to the complete intersection curve C.

It is easy to see that $\mathcal{T}_C$ is, in general, a rank 2 reflexive sheaf, and that $\mathcal{T}_C$ is a subsheaf of the logarithmic sheaves associated to S and T. We say that the curve C is *free with exponents a and b* when $\mathcal{T}_C = \mathcal{O}_{\mathbb{P}^3}(-a) \oplus \mathcal{O}_{\mathbb{P}^3}(-b)$.

Our first remark is that the freeness of C seems to be linked with how degenerate the curve is: one can show that, except for lines, smooth curves are never free, while multiple lines are always free. Our main goal is to provide freeness criteria for complete intersection curves, generalizing some of the known results for the logarithmic sheaves associated to hypersurfaces in $\mathbb{P}^3$.

Here are some of the partial results obtained so far. Let $\Xi_C := V\left(\bigwedge^2 \mathcal{J}_C\right)$, the zero locus of the 2×2 minors of the Jacobian matrix.

- $\mathcal{T}_C$ is locally free if and only if Ξ_C contains no isolated or embedded points.
- when $s = 0$, C is free if and only if $\mathcal{T}_C$ is locally free.
- when $s = 1$ and the surface S is non-singular, C is free if and only if $\mathcal{T}_C$ is locally free.
- if $\mathcal{J}_C$ admits a syzygy that vanishes either in dimension 0, or along a complete intersection curve, then $\mathcal{T}_C$ is free.

Expected results	1. *Understanding when freeness of a pair of polynomials depend only on the ideal they generate.* 2. *Characterization of free curves contained in quadrics (case $s = 1$).*

1.6.4 Lefschetz Properties and moduli space of sheaves on projective spaces

The study of the first cohomology module of a locally free sheaf is relevant to the study of their moduli spaces as can be seeing for instance in [AJTT, ES13, JM10]. Therefore it is important to identify interesting properties for such modules. In this direction, the authors of [FFP] proved that the first cohomology module of any rank 2 locally free sheaf on $\mathbb{P}^2$ has weak Lefschetz Property (WLP), that is, that there exists a linear form $L \in H^0(\mathcal{O}_{\mathbb{P}^2}(1))$ such that the map induced by its multiplication $\times L : H^1(E(t-1)) \to H^1(E(t+1))$ has maximal rank for every integer t.

Once the authors of [HMNW03] proved that the presence of the WLP in a graded Artinian module imposes several restrictions on their possible Hilbert functions and possible Betti numbers, we expect that the presence of WLP in the first cohomology module of locally free sheaves can give new insights in order to understand better their cohomology tables.

Our main goal in this work will be to understand under what conditions the results of [FFP] can be generalized to sheaves on $\mathbb{P}^3$. Our first step will be to study the presence of Lefschetz properties in specific families of sheaves, such as the Instantons sheaves, and generalized nullcorrelation bundles. It is also interesting to note that in [Mig06], the author proved that WLP appears generically in some irreducible component of the Hilbert scheme of zero-dimensional subschemes of $\mathbb{P}^3$, while it fails for every element of another irreducible component. In this direction, it will be interesting to study the presence of WLP for elements in different irreducible components of the moduli space of stable vector bundles on projective spaces.

Expected results	1. *Characterization of families of rank 2 locally free sheaves on $\mathbb{P}^3$ satisfying the Weak Lefschetz Property.* 2. *The study of possible applications for cohomology tables of rank 2 locally free sheaves on $\mathbb{P}^3$.* 3. *Extension of this study for rank 2 properly reflexive and torsion free sheaves on $\mathbb{P}^3$.*

2 Project scope

The main scope of the project is to use the expertise and complementarity of the different research teams and their members in order to obtain new insights in several geometrical settings using methods of geometrical analysis, geometric invariant theory, gauge theory and special, complex and homogeneous geometry.

3 Expected results and guiding tasks

Work package 1	**General theory of harmonic geometric structures**
Members	del Barco, Grama, Rodríguez-Díaz, Fadel, Loubeau, Sá Earp
Objective	Analytic theory of flow and harmonicity conditions in various contexts
Description of work	Task 1.1 *Adapt* [CD90] *for the harmonic heat flow for G-equiv. maps.* Task 1.2 *Computation of harmonicity of generalised complex structures.* Task 1.3 *Situate the integrability condition of an $\{e\}$-structure within the harmonicity chain of sections.*
Expected results	1. *Monotonicity and ε-regularity for G-equiv. maps under* HMHF; *Examples of convergence to a* $\operatorname{div} T = 0$ *solution.* 2. *harmonicity condition for generalised complex structures.* 3. *Relation between torsion and integrability for parallelisms on* $(\mathbb{S}^n, g)$; *Integrability of J on $\mathbb{S}^6$/parallelism on $\mathbb{S}^7$.*
Risk and contingency plan	No general long-time existence ⇝ homogeneous mfds; Integrability too strong ⇝ weaker cond.: nearly-parallel, $(1,2)$-symplectic, cosymplectic.

Work package 2	**Homogeneous harmonic G_2-structures and G_2-HHG$_2$F flow**
Members	del Barco, Grama, Flamencourt, Lauret, Moroianu, Moreno, Saavedra, Sá Earp
Objective	General existence results and construction of examples
Description of work	Task 2.1 *Identify harmonic coclosed* G_2*-struct. in Reidegeld's list.* Task 2.2 *Analysis of the harmonic homogeneous* G_2*-structure cond.* Task 2.3 *Analysis of the associated bracket flow as ODE.* Task 2.4 *Examples on Lie groups: nilpotent, solvable.*
Expected results	1. *Examples of* $\operatorname{div} T = 0$ *homogeneous coclosed* G_2*-structures and* HHG$_2$F *convergence.* 2. *Classify harmonic coclosed* G_2*-structures within Reidegeld's lists and study the solutions of the* HHG$_2$F. 3. *Classify coclosed harmonic* G_2*-structures on solvable Lie groups; Properties of limit structure under Lie bracket* HHG$_2$F. 4. *Study the Killing condition on the total torsion T of harmonic coclosed* G_2*-structures on solvmanifolds.*
Risk and contingency plan	Too hard on general homogeneous spaces ⇝ algebraic constraints.

Work package 3	**Locally conformally Kähler manifolds**
Members	Andrada, Barberis, Chiossi, del Barco, Grama, Grajales, Herrera, Moroianu, Tolcachier
Objective	Construction of new non-Vaisman LCK solvmanifolds
Description of work	Task 3.1 *Modification of methods to produce Kähler Lie algebras.* Task 3.2 *Use Lie theory and algebraic number theory to build lattices.* Task 3.3 *Study sections of holomorphic vector bundles.* Task 3.4 *Study of vector fields on compact LCK manifolds.*
Expected results	1. *New methods to construct unimod. Lie algebras with LCK structures.* 2. *Existence of lattices in some of the corresponding Lie groups.* 3. *Triviality of some holomorphic vector bundles on LCK solvmanifolds.* 4. *Holomorphic vector fields on non-Vaisman LCK solvmanifolds.*
Risk and contingency plan	Examples are scarce ⇝ derive obstructions and prove that known LCK solvmanifolds are the only ones that exist.

Work package 4	**Gauge theory on Kähler and G_2-geometries**
Members	Sá Earp, Oliveira, Clarke, Menet, Gasparim, Rubilar, Suzuki
Objective	Construction of singular twisted connected sums
Description of work	Task 4.1 *Review the deformation theory of reflexive sheaves.* Task 4.2 *Adapt the gluing argument for singular HYM connections.* Task 4.3 *Relax the Hartshorne-Serre construction to reflexive sheaves.* Task 4.4 *Describe Calabi–Yau threefolds obtained via holomorphic gluing of local 3-folds and sheaves on them.* Task 4.5 *Solve the Strominger system on tri-Sasakian related G_2-structures.* Task 4.6 *Classify homogeneous Strominger solutions on Aloff-Wallach mfds.*
Expected results	1. *Gluing conditions for singular* G_2*-instantons over TCS*; *Mass-producing examples of matching pairs of reflexive sheaves.* 2. *New families of G_2 geometries fibered over Calabi–Yau threefolds obtained via holomorphic gluing.* 3. *Solving* G_2*-Strominger syst. on* 3*-Sasakian mfds/reduction to ODE.* 4. *Classifying solutions to* G_2*-Strominger system on Aloff-Wallach mfds.*
Risk and contingency plan	Matching reflexive sheaves are locally-free, so no new examples $\rightsquigarrow$ drop Hartshorne-Serre in favour of Walpuski's approach.

Work package 5	**Kähler reductions, canonical metrics and instantons**
Members	Clarke, Legendre, Menet, Sá Earp, Gasparim, Rubilar, Suzuki.
Objective	Stability of vector bundles and varieties under GIT quotient
Description of work	Task 5.1 *Formulate Minkowski condition for bundles in Kähler reduction.* Task 5.2 *Systematic procedure to calculate slopes and examples.* Task 5.3 *Validate possible synergy with WP 4.* Task 5.4 *Formulate Minkowski condition for varieties.*
Expected results	1. *Embedding stable torsion-free sheaves into G-equiv. stable sheaves*; *Reduction and Hartshorne-Serre construction.* 2. *Minkowski condition for quotients of* G_2*-manifolds.* 3. *Minkowski condition for K-E reduction.*
Risk and contingency plan	No good 'universal' Minkowski cond. $\rightsquigarrow$ quotients in restricted category.

Work package 6	**Moduli spaces in algebraic geometry**
Members	Almeida, Andrade, Correa, Faenzi, Henni, Jardim, Vallès, Vidal Martins
Objective	Holomorphic distributions and stable sheaves on $\mathbb{P}^3$
Description of work	Task 6.1 *Classification of codimension one distributions of degree 2 on* $\mathbb{P}^3$. Task 6.2 *Classification of foliations by of degrees 1 and 2 on* $\mathbb{P}^3$. Task 6.3 *Systematic study of free complete intersection curves on* $\mathbb{P}^3$ Task 6.4 *Study the connectedness of the moduli space of rank 2 sheaves with fixed Chern classes on* $\mathbb{P}^3$. Task 6.5 *Study of families of rank 2 locally free sheaves on* $\mathbb{P}^3$ *satisfying the Weak Lefschetz Property.*
Expected results	1. *Classification of distributions of low degree on* $\mathbb{P}^3$. 2. *Description of connected clusters of irreducible components in the moduli space of stable rank 2 sheaves on* $\mathbb{P}^3$ *with fixed Chern classes.* 3. *Characterization of free complete intersection curves on* $\mathbb{P}^3$. 4. *Classification of rank 2 locally free sheaves on* $\mathbb{P}^3$ *satisfying the Weak Lefschetz Property.*
Risk and contingency plan	If one of the proposed classifications turns out to have too many cases $\rightsquigarrow$ we will look for meaningful hypotheses to make it viable.

B3. Schedule, with main execution stages

Taking into account the funding contribution of each country involved in the Math-AmSud consortium, the planned activities to carry on this project are the following.

First year (2021)

- A conference will be held in Córdoba, Argentina, by the end of the first year.
- Three researchers of the French team will visit Córdoba (15 days each).
- A student of the French team will visit Campinas (15 days).
- One researcher of the Brazilian team will visit Córdoba (20 days).
- One researcher of the Brazilian team will visit France (15 days).
- One member of the Argentinian team will visit France (15 days).
- One member of the Argentinian team will visit Brazil (15 days).
- Two members of the Chilean team will visit Córdoba (a researcher and a PhD student for 15 days each).
- One member of the Chilean team will visit Brazil (a postdoctoral student for 1 month).

Second year (2022)

- A conference in Brest, France, will be held the second year.
- Two Brazilian researchers will visit France (15 and 20 days respectively).
- One member of the French team will visit Brazil (15 days).
- Two members of the French team will visit Chile (a researcher and student 15 days each).
- One member of the Argentinian team will visit Brazil (15 days).
- One member of the Argentinian team will visit France (PhD Student, 15 days).
- Two members of the Chilean team will visit France (a researcher and a postdoctoral student for 20 days each).

Guiding concepts for conferences and missions

The Math-AmSud project will fund the travel expenses of team partners in France, Brazil and Chile to attend the conference in Córdoba, and similarly for the meeting in France. Contributions from other sources will assure the gathering of researchers from most of the participating institution.

The conferences in Córdoba and in France will be on the topics of the present project and will count on the participation of researchers, post-doctoral and PhD students of the research teams, together with specialists from close universities. Researchers attending these meetings will use this opportunity to stay for short periods at the hosting universities in order to visit and interact with the local research group.

During the gathering in France, members will evaluate the state of the collaborations of the teams, the research advances and problems that arose during the development of the project, and will establish the continuation and possible expansion plan of the research teams.

Visiting researchers will participate in seminars and local team meetings in order to make the most of the interchange. In particular, young researchers will benefit from long stays and novel experiences and collaborations in their early stage careers.

Further visits between French and Brazilian researchers are expected, under the already existing CAPES-COFECUB programs. Local sources of funding will be at disposal of the organisation of the aforementioned events, and eventually to partially support other visits and hosting of the partners of present research teams.

B4. Contributions

The University of Campinas, Unicamp, is one of the leading higher-education establishment in South America and its Mathematical department, IMEEC, a prominent research centre in Brazil. With geometry one of its strong suits, Unicamp will contribute to this project on two fronts of the scientific project and act as a connection between Algebraic Geometry and Differential Geometry. M. Jardim is a leading specialist in Moduli Spaces and has collaborated with researchers in Brazil (UFF, UFSC, UFMG) and France. He will lead the Algebraic Geometry side of the project. H. Sa Earp works in Differential Geometry, more precisely in Special Geometries and Gauge Theory, and has a wide network of collaborations (USA, UK, France) and several PhD students involved in the project. His investigations often lead him to questions of Algebraic Geometry and, for example, his recent works on G_2-structures on homogeneous spaces, with A. Moreno and J. Saavedra, initiated collaborations with J. Lauret of the FAMAF (Córdoba), a research subject where several other members work, notably L. Grama, V. del Barco and A. Moroianu. In this project, Unicamp will be able to play a pivotal role by bridging collaborations between South American countries and France.

The FAMAF of Córdoba University, boasts a world-leading research group in homogeneous geometry and representation theory, which stands at the cross-road of the scientific project. With this cooperation, members of FAMAF will start new collaborations on both Algebraic and Differential Geometry problems, invite researchers in Córdoba and travel abroad. A workshop gathering all members will be organised in Córdoba as part of the project.

The LMO at Orsay is one of the most important and largest Mathematics research unit in France (with Fields medals, academicians and several ERC grants) and is probably on-par with the best centres in the world. The research pair of V. del Barco and A. Moroianu have been working on homogeneous geometry (in Differential Geometry) for several years and will be an important contributor to the study of G_2-structures on homogeneous manifolds, using the idea of Killing tensors, and complement the efforts of the FAMAF and Unicamp groups in the search for solutions to the harmonic flow. With its brand new facilities, the LMO will be able to host visitors from the project and provide the best working conditions.

The LMBA is the Maths research unit of the University of Brest and hosts some fifty researchers, including a Differential Geometry team of a dozen people. The two researchers involved in the project work in the larger subject of harmonic maps and have collaborated with Unicamp over the last three years. Their input will mainly be on the Differential Geometry aspects of structures on manifolds. The LMBA has facilities to welcome visitors from the project.

The Institut de Mathématiques de Toulouse is one of the most important Mathematical research centres in France, both in size and in quality. It covers a broad spectrum of subjects and is particularly well-known for its geometry group. E. Legendre is notably well-established in the field of Special Geometries and will be an essential participant of the project, collaborating with Brazilian members in Unicamp, UFRJ and UFF, and Argentinian specialists of homogeneous geometry. The Institut Fourier in Grenoble is a leading research unit in Mathematics, with a historically strong track-record in geometry and a world-wide reputation. G. Menet stands at the junction of Algebraic Geometry and Differential Geometry and is an essential cog of the project, interacting with H. Sa Earp in Unicamp and J. Lauret in UNC. Both institutions host a very large number of visitors every year and have excellent equipment and resources to welcome members from the project.

The department of Mathematics of the Universidad Católica del Norte in Antogafasta (UCN), Chile, is the leading institution for the northern Andean region and has become a pole for research in Mathematics. E. Gasparim is a specialist of Algebraic Geometry, more specifically of moduli spaces, and of Lie group theory, with an international network of coworkers (Brazil, USA, Italy, UK). She has collaborated with several other members of the project, notably L. Grama and M. Jardim, on more than five articles.

The UFRJ and UFF are two of the most important universities in Brazil with substantial Mathematics departments and international reputations. The project members in Rio de Janeiro are experts of Special Geometries and, over the years, have woven strong ties with researchers in Campinas, Toulouse and Brest. These existing collaborations will aggregate other participants of the Math-AmSud programme around new research projects and allow cooperation to start straightaway.

A member of the Maths department in Florianopolis at UF Santa Catarina, one of the top universities in Brazil, A. Henni has collaborated for many years with M. Jardim on Moduli Spaces and sheaf deformations on projective varieties, a subject that interacts with several constructions of G_2-manifolds.

Members at UFMG and UFF have also been involved in previous research projects around M. Jardim on holomorphic foliations and moduli spaces. Long-time collaborators of M. Jardim and the Brazilian team, J. Vallès (Pau University) and D. Faenzi (University of Bourgogne) are specialists of hyperplane arrangements in projective spaces and holomorphic foliations, and have participated in several Franco-Brazilian research projects. Their expertise will enable the team to expand the scope of its investigation of algebraic conditions of Gauge Theory towards the holomorphic foliations and distributions that will appear in the moduli spaces of homogeneous structures. The research units in Pau and Dijon are well-established centres and will be able to host visitors from the project.

B5. Regional Aspects

This project is based on a double cooperation: Scientific between Algebraic Geometry and Differential Geometry; Geographic along the axis Argentina-Brazil-Chile-France.

The present project is a natural escalation of the several existing bi-national co-operations, between Argentina-Brazil, Chile-Brazil, France-Brazil and Argentina-France, to a multi-national network inside South America together with France. The pursued enlarged network will be a support that will strengthen the leaderships of the research groups in their own countries and boost their regional and international recognition.

The IMECC at Unicamp stands at the cross-road of this programme with its expertise in Moduli Spaces and Gauge theory, and its regional and transatlantic research partnerships. IMECC will therefore act as go-between and pivot of the project around the theme of homogeneous spaces and their harmonic flows.

The principal objective of this scheme is to enable researchers in South America, in particular young mathematicians, to collaborate and stay in France for extended periods of time, and it will build upon exchanges of PhD and post-doctoral students. This will notably help developing their current research projects but also lay the foundations for future long-term co-operations.

The collaboration between IMECC and FAMAF (Córdoba) on the harmonic flow on homogeneous spaces will co-involve mathematics laboratories in Brest and Orsay. Specifically, the research group focused on the general theory of harmonic geometric structures, composed of Sá Earp, Grama, Loubeau and del Barco, will feed information to the cluster dedicated to homogeneous harmonic G_2-structures and G_2-flow, made-up of Sá Earp, Lauret, Moreno, Saavedra, Grama, del Barco and Moroianu, which in turn will provide some explicit behavioural examples to feed-back into the general theory and shed light.

Andrada, Barberis and Moroianu from FAMAF and UPSaclay have been collaborating the last few years obtaining important results in conformal geometry. This Math-AmSud project will enable them to include specialists from Unicamp and UFF to further develop their techniques.

The Chilean team will merge with the French-Brazilian side working on Gauge theory and Kähler reductions, gathering researchers from Campinas and Rio de Janeiro on the one side and from Toulouse and Grenoble on the other side, enhancing some preexisting collaborations between Gasparim's team and the Algebraic Geometry and Lie theory research groups at Unicamp. In particular, E. Gasparim and V. del Barco have both collaborated with the Lie theory group at Campinas, and foresee future projects in the study of homogeneous invariant structures on flag manifolds.

On the Algebraic Geometry side, the long-standing cooperation between Brazilian universities (Unicamp, UFSC, UFMG and UFF) and French institutions (Univ. Bourgogne in Dijon, IF in Grenoble and Univ. Pau) on Moduli Spaces and holomorphic foliations will open up to new partners, in particular FAMAF and UCN, to build up new collaborations and research projects between Argentina, Brazil, Chile and France.

New research questions will appear from this trade-off and sub-teams will be set-up to tackle them, generating novel interactions among members of the project and leading to join publications, reciprocal invitations and co-supervisions.

These exchanges will create a new synergy around a theme common to all associated countries and induce an integrated research activity enabling further interactions with other research groups in Europe.

To stimulate these discussions and exchanges, two meetings will be organised to gather all project participants and initiate the cross-fertilization of ideas. The first year, a workshop will be held at FAMAF (Córdoba, Arg.), towards the end of 2021, and the second year a meeting at UBO (Brest, France) is expected.

B6. Institutions and CVs of coordinators

See next page.

ANNEX

DEL BARCO Viviana

1/ Personal data:

Name: Viviana del Barco

Birth date: July 31st, 1983

Professional address (with telephone and e-mail):

viviana.del-barco@math.u-psud.fr

Bureau 2O2

Département de Mathématiques – Bâtiment 307

Faculté des Sciences d'Orsay

Université Paris-Saclay

F-91405 Orsay Cedex

+33 (0) 1 69 15 79 56

Current job title and size of the research group:

- ATER at Université Paris Saclay – France. Research group Topology and Dynamics : 30 researchers.
- Assistant Researcher at CONICET, Consejo Nacional de Investigaciones Científicas y Técnicas – Argentina (on leave).

2/ Highest obtained degree (with indication of place and date)

Doctorate in Mathematics, Universidad Nacional de Rosario, Argentina. March 2012.

3/ Professional activity in the last 5 years

ATER at UPSaclay – France, since 2019.

Assistant Researcher at CONICET – Argentina, since 2015

Postdoctoral Researcher at UNICAMP – Brazil (2016-2019)

Research visit to UPSaclay – France (2018)

4/ Other duties/ positions --

5/ Awards, fellowships and external recognition

- FAPESP – Postdoctoral fellowships (2016-2019)
- FAPESP – BEPE fellowship (2018) to visit Laboratoire de Mathématiques d'Orsay, Université Paris Sud.

- Seal of Excellence - Marie Sklodowska-Curie Actions. SkewTorOnLie Project Submitted under the Horizon 2020 MSC actions call H2020-MSCA-IF-2018 obtained 92.2% score. This quality label is awarded to all proposals submitted to the MSCA Individual Fellowships Call that scored 85% or more but could not be funded from the call budget.

6/ Ongoing funded research projects with dates, titles, sources of funding

01/2018-12/2020 – UNR Argentina. Lie groups in pseudo and sub-Riemannian geometry. Participation as collaborator. General coordinator: Silvio Reggaini

7/ Projects approved in the least 5 years

06/2019 – ICTP-INdAM, Italy. Research in Pairs. Research visit Università degli Studi di Milano-Biccoca (1 month), 2500eur.

2014 - 2016 – PICT Foncyt, Argentina. Geometric structures in homogeneous spaces. Participation as coordinator group. General coordinator: Gabriela Ovando. 150000 Ars.

8/ Publications

8.1 – Highlight the most important publications related to the project theme

V. del Barco and A. Moroianu. "Killing forms on 2-step nilmanifolds". J. Geom. Anal. (2019) doi:10.1007/s12220-019-00304-1

V. del Barco and A. Moroianu. "Symmetric Killing tensors on nilmanifolds". To appear in Bull. Soc. Math. France. (2018) arXiv:1811.09187

V. del Barco and A. Moroianu. "Higher degree Killing forms on 2-step nilmanifolds". Preprint. 2020 arXiv:2002.01208

V. del Barco and L. Grama. "On generalized G 2 -structures and T -duality." J. Geom. Phys. 132 (2018), pp. 109–113 doi:10.1016/j.geomphys.2018.05.021

8.2 – Publications in cooperation with the project partners

V. del Barco and L. Grama. "On generalized G 2 -structures and T -duality." J. Geom. Phys. 132 (2018), pp. 109–113 doi:10.1016/j.geomphys.2018.05.021

6. V. del Barco, L. Grama, and L. Soriani. "T -duality on nilmanifolds." J. High Energy Phys. 2018.5 (2018), p. 25 doi:10.1007/JHEP05(2018)153

V. del Barco and A. Moroianu. "Killing forms on 2-step nilmanifolds". J. Geom. Anal. (2019) doi:10.1007/s12220-019-00304-1

V. del Barco and A. Moroianu. "Symmetric Killing tensors on nilmanifolds". To appear in Bull. Soc. Math. France. (2018) arXiv:1811.09187

V. del Barco and A. Moroianu. "Higher degree Killing forms on 2-step nilmanifolds". Preprint. 2020 arXiv:2002.01208

9/ Theses oriented and post-doctoral fellows supervised. --

ANNEX

ANDRADA, Adrián Marcelo

1/ Personal data

Name: Adrián Marcelo Andrada

Birth date: 26 November 1976

Professional address (with telephone and e-mail):

Facultad de Matemática, Astronomía, Física y Computación
Universidad Nacional de Córdoba
Av. Medina Allende S/N - Ciudad Universitaria
X5000HUA - Córdoba, Argentina
Tel: +54 351 5353701 ext. 41351
E-mail: andrada@famaf.unc.edu.ar

Current job title and size of the research group:

Associate Professor

Differential Geometry Group: has 20 members.

2/ Highest obtained degree (with indication of place and date)

Doctor en Matemática (PhD in Mathematics) - Universidad Nacional de Córdoba, December 2003.

3/ Professional activity in the last 5 years

- Professor at Facultad de Matemática, Astronomía, Física y Computación - Universidad Nacional de Córdoba.
- Researcher at the CONICET (National Council for Scientific Researh and Technology, from Argentina). Still ongoing

4/ Other duties/ positions

5/ Awards, fellowships and external recognition

6/ Ongoing funded research projects with dates, titles, sources of funding

- Research project: *Estructuras geométricas en variedades localmente homogéneas*. Director: María Laura Barberis. From January 2019 to December 2022. Source of funding: Universidad Nacional de Córdoba (Argentina).

7/ Projects approved in the least 5 years

- Research project: Estructuras geométricas invariantes en solvariedades. Director: Adrián Marcelo Andrada. From January 2016 to December 2017. Source of funding: Universidad Nacional de Córdoba (Argentina).

8/ Publications

8.1 – Highlight the most important publications related to the project theme

- A. Andrada, A. Fino, L. Vezzoni, *A class of Sasakian 5-manifolds*, Transform. Groups 14 (2009), 493-512.
- A. Andrada, M. L. Barberis, I. Dotti, *Classification of abelian complex structures on six-dimensional Lie algebras*, J. London Math. Soc. (2011), 232-255.
- A. Andrada, M. L. Barberis, I. Dotti, *Invariant solutions to the conformal Killing-Yano equations on Lie groups*, J. Geom. Phys. 94 (2015), 199-208.
- A. Andrada, M. Origlia, *Lattices in almost abelian Lie groups with locally conformally Kähler or symplectic structures*, Manuscripta Math. 155 (2018), 389-417.
- A. Andrada, M. Origlia, *Vaisman solvmanifolds and relations with other geometric structures*, accepted for publication in Asian J. Math.

8.2 – Publications in cooperation with the project partners

- A. Andrada, M. L. Barberis, A. Moroianu, *Conformal Killing 2-forms on 4-dimensional manifolds*, Ann. Global Anal. Geom. 50 (2016), 381-394.

9/ Theses oriented and post-doctoral fellows supervised

9.1 – Finished/defended in the last 5 years

- Doctoral thesis: Marcos Origlia (defended in March 2017).
- Post-doctoral fellow: Marcos Origlia (April 2017 - March 2019)
- Post-doctoral fellow: Andrea Cecilia Herrera (April 2018 - March 2020).

9.2 – Ongoing

- Doctoral thesis: Alejandro Tolcachier (since April 2019).

HENRIQUE N. SÁ EARP

ORCID 0000-0003-0475-4494 • ResearcherID G-4427-2015
Scopus 8707619100 • Google Scholar • Lattes CV

1/ PERSONAL DATA

Associate Professor of Mathematics • Born 16 January 1981

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(+55) 21 9 82 72 1234

Research groups		Faculty	Post-Docs	PhD	Masters	Σ
Gauge Theory and Algebraic Geometry	GTAG	3	3	6	1	13
Differential Geometry	GeoDif	4	1	6	5	16
Geometry and Physics	G+F	9	1	14	2	26

2/ HIGHEST DEGREES

2019	***Livre-docência* - Associate professorship**, University of Campinas (Unicamp)	2004-2009	**PhD**, Imperial College London Supervisor: Sir Simon K. Donaldson

3/ PROFESSIONAL HISTORY [5 YEARS]

Institute of Mathematics, Statistics and Scientific Computing (IMECC), Unicamp

2020-	Associate Professor	2016-2019	Adjoint Professor II

4/ OTHER DUTIES AND POSITIONS

IMECC, Unicamp

2010–2015	Adjoint Professor (MS3.1)
2009-2010	Post-doc (Fapesp)

Centre for Quantum Computing, University of Cambridge

2004	Research Studentship (Royal Society)

5/ AWARDS, FELLOWSHIPS AND EXTERNAL RECOGNITION

2015-	**Brazilian National Research Council (CNPq)**: Research productivity, level PQ2

6/ ONGOING FUNDED RESEARCH PROJECTS (AS PRINCIPAL INVESTIGATOR)

2019-2024	**Fapesp - Thematic project**: Gauge theory and Algebraic Geometry, with M. Jardim (Unicamp) + 2 AIs.
2018-2021	**CAPES-Cofecub - binational collaboration**: Unicamp-Unifesp-UFRJ-UFF & Brest-Nantes, with E. Loubeau (Université de Bretagne Occidentale, France) + 8 AIs.
2019-2021	**Royal Society Newton Mobility Award**, with J. Lotay (Oxford University).

7/ FORMER FUNDED RESEARCH PROJECTS (AS PRINCIPAL INVESTIGATOR) [5 YEARS]

Fev/2019	**Santander Universidades - Bilateral collaboration**, with J. Lauret (Univ. Nacional de Córdoba, Argnetina).
Mar-Abr/2017	**Institut Français du Brésil - Chaire Franco-Brésilienne**: E. Loubeau (UBO) at IMECC-Unicamp.
2016-2017	**Faepex (Unicamp) - 'Session *Trends in Geometry and Topology*'**, II Brazilian Congress of Young Mathematicians.
2016-2017	**Fapesp & MIT Seed Fund - Bilateral collaboration**, with T. Walpuski (MIT, USA).
2016-2017	**Fapesp - Visiting scholar**: D. Tsonev (Federal Univ. Amazonas) at IMECC-Unicamp.
2015-2020	**Fapesp - Regular Research Grant.**

8/ Highlighted publications [project **partners**]

2020	Calvo-Andrade, O., **Rodríguez, L.** & **HSE**, *Gauge theory and* G_2*-geometry on Calabi-Yau links*, Rev. Mat. Iberoam. (online first).	[SJR Q1]
2019	**Loubeau, E.** & **HSE**, *Harmonic flow of geometric structures*, preprint. 1907.06072	
2019	**Fadel, D.** & **HSE**, *Introduction to gauge theory in higher dimensions*, book to appear, IMPA-Springer.	
2019	**Menet, G.**, Nordström, J. & **HSE**, *Construction of* G_2*-instantons via twisted connected sums*, to appear in Math. Res. Lett. 1510.03836	[SJR Q1]
2019	**Moreno, A. J.** & **HSE**, *Explicit soliton for the Laplacian co-flow on a solvmanifold*, Sao Paulo J. Math. Sci., 'Proceedings of the satellite conference to ICM2018 on Differential Geometry'.	
2019	**Moreno, A. J.** & **HSE**, *The Weitzenböck Formula for the Dirac-Fueter operator*, to appear in: Commun. Anal. Geom. 1701.06061	[SJR Q1]
2018	**HSE**, *Current progress on* G_2*–instantons over twisted connected sums*, to appear in: Fields Inst. Commun., 'Lectures and Surveys on G_2–manifolds...'. 1812.04664	[SJR Q3]
2017	**Jardim, M. B.**, **Menet, G.**, Prata, D. M. & **HSE**, *Holomorphic bundles for higher dimensional gauge theory*, Bull. London Math. Soc. **49** (1).	[SJR Q1]
2015	**HSE** & Walpuski, T., G_2*–instantons on twisted connected sums*, Geom. Topol. **19** (3).	[SJR Q1]
2015	**HSE**, G_2*–instantons on asymptotically cylindrical manifolds*, Geom. Topol. **19** (1).	[SJR Q1]

8/ Academic supervision

8.1/ Former

Post-docs (2)

2016-2017 Dragomir Tsonev, *Open problems in curvature and holonomy.*

2015-2017 Lázaro O. Rodríguez, *Topological invariants in* G_2*-geometry.*

PhD (4)

2016-2020 Luis E. Portilla, *Contact instantons on Sasakian 7-manifolds.*

2016-2020 Daniel G. Fadel, *Blow-up loci of* G_2*-monopoles.*

2015-2019 Pedro M. M. de Paula, *Yang-Mills-Higgs metrics over ACyl manifolds.*

2015-2019 Andrés J. Moreno Ospina, *Algebraic methods in* G_2*-geometry.*

Masters (5)

2017-2018 Augusto C. Pereira, *Completeness of the ADHM construction.*

2016-2017 Julieth P. Saavedra, *Flows of homogeneous* G_2*-structures.*

2015-2016 Daniel G. Fadel, *Blow-up loci of instantons in higher dimensions.*

2015-2016 Luis E. Portilla. *Fredholm theory and instantons in dimension 4.*

2014-2015 Andrés J. Moreno. *The Fueter operator and deformations of associative submanifolds.*

Masters in Mathematical Education - ProfMat (1)

2016-2018 Diana T. Amaro. *Elements of topology in the school curriculum.*

8.2/ Ongoing

PhD (1)

2017-2021 Julieth P. Saavedra. *Solitons of the Laplacian coflow of* G_2*-structures.*

Masters (1)

2020-2022 Luis Henrique Lara. *Laplacian eigenvalues on homogeneous spaces.*

CV Elizabeth Gasparim

1/ Personal data

Name: Elizabeth Terezinha Gasparim

Birth date: 20-05-1963

Professional address:

Universidad Católica del Norte, Antofagasta, Chile,

etgasparim@gmail.com, phone: +56972834584.

Current job title and size of the research group:

Professor, research group 4 members.

2/ Highest obtained degree: PhD The University of New Mexico 1995.

3/ Professional activity in the last 5 years:

Coordinator of the Network NT8 for cooperation between Chile, Brasil, Colombia, and Peru, supported by OEA-ICTP.

Member of the direction of the Virtual Institute for Geometry and Physics - joint with Mexico and USA.

4/ Other duties/ positions:

Coordinator of the Affiliated Institute - Facultad de Ciencias UCN - ICTP.

Member of organising/ research committees of CIMPA schools in Peru and Colombia.

5/ Awards, fellowships and external recognition:

Simons Associateship ICTP.

6/ Ongoing funded research projects with dates, titles, sources of funding:

Cooperation in Algebraic Geometry with Prof. Ulf. Persson, Sweden, funded by Conicyt.

Lefschetz Fibrations, Lie Groupoids and Noncommutative Geometry, with Prof. Cristián Ortiz, Brazil, funded by ANID and Fapesp.

7/ Projects approved in the least 5 years:

University funded yearly research projects, UCN Chile (3 times).

Network NT8 OEA- ICTP.

Summer/Winter Schools and Workshops in Geometry (5 times) funded by OEA and the local institutions, and held at ICTP (Italy), Universidad de Antioquia (Colombia), Universidade Estadual de Maringá (Brazil), Universidad Nacional San Antonio Abad del Cusco (Peru), Universidad Católica del Norte (Chile).

8/ Publications

8.1 – Highlight the most important publications related to the project theme:

1. **Deformations of noncompact Calabi--Yau manifolds, families and diamonds, with F. Rubilar, to appear in Contemporary Math. (2020)**
2. **Classical deformations of local surfaces and their moduli of instantons, with S. Barmeier, J. Pure and Applied Algebra. 223 no. 6, (2019), 2543–2561.**
3. **Topological String Partition Function on Generalised Conifolds, with B. Suzuki, A. Torres-Gomez and C. Varea, Journal of Mathematical Physics, 58 (2017) 1–16.**

8.2 – Publications in cooperation with the project partners: (there are several I give the most recent)

1. **A Lie theoretical construction of a Landau-Ginzburg model without projective mirrors, with Ballico, E.; Barmeier, S.; Grama, L.; San Martin, L. A. B. Manuscripta Math. 158 (2019), no. 1-2, 85–101.**

9/ Theses oriented and post-doctoral fellows supervised

9.1 – Finished/defended in the last 5 years

Bruno Sukuzi Ph.D. UCN Chile 2019.

Severin Barmeier Ph.D. UCN and University of Münster 2018.

Alex Poveda Master UCN Chile 2018.

Alfredo Nisperuza Master UCN Chile 2017.

Francisco Rubilar Master UCN Chile 2017.

Carlos Bassani Varea (co-supervisor) Master Unicamp Brazil 2016.

9.2 – Ongoing

Francisco Rubilar, current Ph.D. student, UCN Chile.

B7. Additional information

List all the complementary funding expected or already obtained.

- 2019–2022 CAPES-COFECUB French-Brazilian collaboration “Moduli spaces in algebraic geometry and applications”. Led by M. Jardim and D. Faenzi.
- 2018–2021 CAPES-COFECUB French-Brazilian collaboration “Special geometries and gauge theory”. Led by H. Sá Earp and E. Loubeau.
- 2019–2024 FAPESP thematic project. Led by M. Jardim and H. Sá Earp.
- 2019–2022 University of Córdoba research project. Led by M.L. Barberis.
- Simons Associateship ICTP of E. Gasparim.
- FAPESP-ANID Chilean-Brazilian Project, between UCN-Universidade de São Paulo, leaded by E. Gasparim.
- Affiliated Institute of ICTP at UCN, funding the research team at UCN.
- 2021–2025 FAPESP-ANR proposal under evaluation.

Experience of the coordinators in similar projects.

Elizabeth Gasparim is nowadays leading an ICTP Simons Associateship grant for research collaborations and visits to the ICTP in Trieste, Italy. She has participated as co-investigator of a DPI-Conicyt project, Chile, which aims to establish and develop international collaboration networks between Chile and the UK. Gasparim has also taken part in an NT8-ICTP cooperation grant gathering researchers from Chile, Peru, Colombia and Brazil. She has been responsible for several Conicyt projects and received funds from CNPQ and Fapesp supporting her research.

Henrique Sá Earp is principal investigator leading a Fapesp Thematic project, joint with M. Jardim, a CAPES-Cofecub France-Brazil collaboration, with E. Loubeau, and a Royal Society Newton Mobility Award, from Oxford University. Over the last five years he has also received funds from public and private institutions to carry on bilateral research projects and supervise students, such as for instance, Institut Français du Brésil, Fapex, Fapesp-MIT. In particular, Santander Universidades funded a bilateral project carried out in collaboration with J. Lauret.

Adrián Andrada is currently associated investigator of a research project from UNC and has been the principal investigator of such projects in 2016-2017. Through these projects, the university funds the mobility of local and visiting researchers. Over the years, he has been the supervisor of several CONICET PhD and Post-doctoral fellowships.

Viviana del Barco has received funding from Fapesp and ICTP-INdAM (Italy) for research stays of 1 year (2018) in Paris, France and 1 month (06/2019) in Milan, Italy, respectively. She has participated as part of the coordinator group of a PICT-Foncyt research project in Argentina in 2014-2016. Currently she participates as collaborator of a UNRosario research project.

Present main activities and their relationship with the project's main goal.

The general coordination of the project will be carried out by Viviana del Barco as part of the French team but also as a CONICET researcher. She is a specialist in homogeneous geometry and has close collaboration with Argentinian, Brazilian and French mathematicians. Notably, she did her PhD with Isabel Dotti from FAMAF who is nowadays Emeritus and founder of the research team; V. del Barco also spent two years as postdoctoral researcher at Unicamp and she is now working at UPSaclay. Within the team, she actively collaborates with Moroianu and Grama on the interaction of (generalized) G_2-structures with Killing tensors on Lie groups.

Henrique Sá Earp is Associate Professor at the University of Campinas, in the state of São Paulo. He is part of the Gauge Theory and Algebraic Geometry (GTAG) group together with M. Jardim, and currently manages one of Brazil-France collaboration, together with E. Loubeau, in topics related to gauge theory and special geometry. Their research project involves the participation of team members in Rio de Janeiro, and has promoted several co-operations between UBO, UFRJ, UFF and UNICAMP.

Sá Earp in particular collaborates with Rodríguez from UFRJ, Menet from Grenoble, former postdocs at GTAG. He supervised Moreno and Fadel, postdocs of the research team on harmonic geometric structures and G_2-geometry. The former nowadays visiting UBO and the latter now working with G. Oliveira at UFF.

Team members Sá Earp and Lauret have recently put up the AmSul/AmSud Geometry Webinar, together with other geometers from South America. This virtual seminar aims to establish contact with several research groups and mathematicians from Latin America.

The differential geometry group at FAMAF is composed of leading experts in homogeneous structures, Lie group actions and invariant special geometry. A. Moroianu visited Córdoba on two occasions which led to collaborations with A. Andrada and M. L. Barberis, and joint publications on conformal Killing tensors. Their expertise will ensure the interaction of the team working on locally conformally Kähler geometry with the team on special G_2-structures related to Killing tensors and invariant structures on Lie algebras, which count with the participation of Brazilian partners.

Elizabeth Gasparim is a specialist of algebraic geometry and its interactions with differential geometry and Lie theory. She has collaborated with team members at Unicamp, namely M. Jardim and L. Grama, but also with other researchers from this institution, where she spent three years as visiting professor (2011-2014).

Perspectives of continuing collaboration after project financing is over.

The present Math-AmSud project will be the opportunity to consolidate bi-national collaborations and to escalate them towards a multinational network. The establishment of new research lines of interest, researcher exchanges and the formation of students will assure the continuation of the cooperation. The topics involved in the project, within the research area of Geometry and Topology, show clear interaction between more than one country-team, synergy which will be the base of future projects inside the present proposed network.

B8. Public and private support obtained related to the project

Participation in a previous STIC-AmSud or MATH-AmSud project? NO

Other public support in the past (ECOS, COFECUB, CNRS, European Union, etc.): –

Other private support in the past:

Prospects for public or private support in the future:

Part of the Brazilian and French teams are at this moment applying for a FAPESP-ANR project whose results will be published by the end of 2020. If approved, these groups will count with additional funding for PhD and postdoctoral fellowships, for short visits and conferences, which will enhance the interchange between Brazil and France, giving a complementary support to the present Math-AmSud project.

C. Project Budget

Project title: Geometric structures and moduli spaces.
Participating institutions: UNC (Argentina), UCN (Chile), UNICAMP, UFRJ, UFF, UFSC, UFMG (Brazil), UPSaclay, UBO, IMT, IF, IMAG (France)

The MATH-AmSud program funds travel expenses (air tickets and per diem) to researchers in research missions and workshops.

C1. First year (2021)

Planned missions

Researcher	Status	Institution	Origin	Destination	Planned date	Duration	Travel (€)	Per diem (€)	Mission funding institution	Mission objectives
A. Henni	Senior	UFSC	Florianópolis, Br	Dijon, Fr	September	15 days	900	1150	CAPES	RM
H. Sá Earp	Senior	Unicamp	Campinas, Br	UNC, Arg	October	20 days	300	2300	CAPES	RM
Post-doc	Junior	Unicamp	Campinas, Br	Brest, Fr	Jan-Oct	10 months	900	21.000	CAPES	SM
E. Loubeau	Senior	UBO	Brest, Fr	UNC, Arg	October	15 days	1100	500	CNRS	RM
A. Moroianu	Senior	UPSaclay	Orsay, Fr	UNC, Arg	October	15 days	1100	500	CNRS	RM & W
D. Faenzi	Senior	UB	Dijon, Fr	Unicamp & UFMG, Br	April	15 days	900	500	MEAE	RM
H. Bibi	Junior	UBO	Brest, Fr	Unicamp, Br	September	15 days	900	500	MEAE	SM
A. C. Herrera	Senior	FAMAF	UNC, Arg	UPSaclay, Fr	June	15 days	1100	1100	MINCYT	RM
A. Andrada	Senior	FAMAF	UNC, Arg	Unicamp, Br	August	15 days	300	500	MINCYT	RM
E. Gasparim	Senior	UCN	Antofagasta, Ch	UNC, Arg	October	15 days	300	495	ANID	RM & W
F. Rubilar	Junior	UCN	Antofagasta, Ch	UNC, Arg	October	15 days	300	495	ANID	SM & W
B. Suzuki	Junior	UCN	Antofagasta, Ch	Unicamp, Br	March	1 month	300	990	ANID	SM

RM: Research mission. W: Workshop. SM: Study mission (PhD or Postdoc).

C2. Second year (2022)

Planned missions

Researcher	Status	Institution	Origin	Destination	Planned date	Duration	Travel (€)	Per diem (€)	Mission funding institution	Mission objectives
C. Almeida	Senior	UFMG	Brazil	UPPA, France	September	15 days	900	1150	CAPES	RM
L. Rodriguez-Diaz	Senior	UFRJ	Brazil	Brest, France	June	20 days	900	1600	CAPES	RM
J. Vallès	Senior	UPPA	Pau, Fr	Unicamp & UFMG, Brazil	November	15 days	900	500	CNRS	RM
V. del Barco	Senior	UPSaclay	Orsay, Fr	UCN, Chile	April	15 days	1100	500	CNRS	RM
PhD Student/ Postdoc	Junior		France	UCN, Chile	May	15 days	900	500	MEAE	SM
A. Andrada	Senior	FAMAF	UNC, Arg	UFF, Brazil	August	15 days	300	500	MINCYT	RM
A. Tolcachier	Junior	FAMAF	UNC, Arg	UPSaclay, Fr	May	15 days	1100	1100	MINCYT	SM
E. Gasparim	Senior	UCN	Antofagasta, Ch	UPSaclay, Fr	June	20 days	1000	1500	ANID	RM & W
F. Rubilar	Junior	UCN	Antofagasta, Ch	UPSaclay, Fr	June	20 days	1000	1500	ANID	SM & W

RM: Research mission. W: Workshop. SM: Study mission (PhD or Postdoc).

CONSOLIDATED BUDGET: Year 1
Funding requested to the MATH-AmSud Program
Estimated costs (€)

	A. Travel costs (air tickets)	B. Maintenance costs (per diem)	TOTAL
MEAE France	1800	1000	2800
CNRS France	2200	1000	3200
Ministerio de Ciencia Tecnología e Innovación de Argentina (Mincyt)	1400	1600	3000
CAPES Brazil	1200	1850	3050
ANID Chile	900	1980	2880
Total requested funding to MATH-AmSud	7500	7430	14930
Other funding[1]	2000	1000	3000
TOTAL	9500	8430	17930

CONSOLIDATED BUDGET: Year 2
Funding requested to the MATH-AmSud Program
Estimated costs (€)

	A. Travel costs (air tickets)	B. Maintenance costs (per diem)	TOTAL
MEAE France	900	500	1400
CNRS France	2000	1000	3000
Ministerio de Ciencia Tecnología e Innovación de Argentina (Mincyt)	1400	1600	3000
CAPES Brazil	1800	2750	4550
ANID Chile	2000	3000	5000
Total requested funding to MATH-AmSud	8100	8850	16950
Other funding[1]	1000	2000	3000
TOTAL	9100	10850	19950

[1] These funds correspond to the additional funding that the project counts on, please see the detailed sources above.

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