

Comparação de estimadores, sob a ótica frequentista, no modelo de Bernoulli

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Relembrando o Exemplo 1 (verossimilhança Bernoulli) (aqui)

- Todos os resultados analíticos devem ser obtidos ou replicados.
- Cenário: $X_i|\theta \stackrel{i.i.d.}{\sim} \text{Bernoulli}(\theta), i = 1, 2, \dots, n$.
- Priori: $\theta \sim \text{Beta}(a, b)$.
- EMV: $\hat{\theta} = \bar{X}$.
- Posteriori: $\theta|\mathbf{x} \sim \text{Beta}(n\bar{x} + a, n(1 - \bar{x}) + b)$.

Relembrando o Exemplo 1 (verossimilhança Bernoulli) (aqui)

- Esperança e moda a posteriori (* a mediana a posteriori não possui forma analítica exata, somente aproximada):

- $\hat{\theta}_{EAP} = \frac{n\bar{X} + a}{n + a + b}$

- $\hat{\theta}_{MAP} = \frac{n\bar{X} + a - 1}{n + a + b - 2}$

- Variância e desvio-padrão à posteriori:

- $\mathcal{V}(\theta|\mathbf{x}) = \frac{(n\bar{x} + a)(n(1 - \bar{x}) + b)}{(n + a + b)^2(n + a + b + 1)}$

- $DP(\theta|\mathbf{x}) = \sqrt{\frac{(n\bar{x} + a)(n(1 - \bar{x}) + b)}{(n + a + b)^2(n + a + b + 1)}}$

Medidas de precisão frequentistas dos estimadores

- Estimador de MV (* B : vício, EQM : erro quadrático médio).

- $\mathcal{E}(\hat{\theta}_{MV}) = \theta.$

- $\mathcal{V}(\hat{\theta}_{MV}) = \frac{\theta(1-\theta)}{n}.$

- $B(\hat{\theta}_{MV}) = 0.$

- $EQM(\hat{\theta}_{MV}) = \frac{\theta(1-\theta)}{n}.$

- $EAP(\hat{\theta}_{EAP}).$

- $\mathcal{E}(\hat{\theta}_{EAP}) = \frac{n\theta + a}{n + a + b}.$

- $\mathcal{V}(\hat{\theta}_{EAP}) = \frac{n\theta(1-\theta)}{(n + a + b)^2}.$

- $B(\hat{\theta}_{EAP}) = \frac{a - \theta(a + b)}{n + a + b}.$

- $EQM(\hat{\theta}_{EAP}) = \frac{n\theta(1-\theta) + [a - \theta(a + b)]^2}{(n + a + b)^2}.$

Medidas de precisão frequentistas dos estimadores (cont.)

- $\text{MAP}(\hat{\theta}_{\text{MAP}})$.

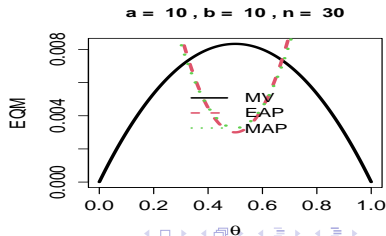
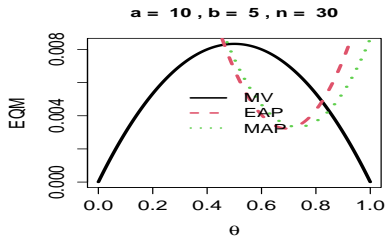
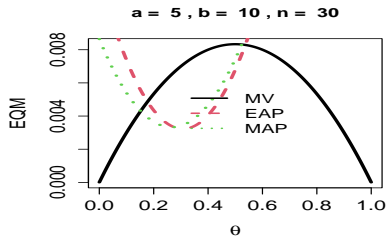
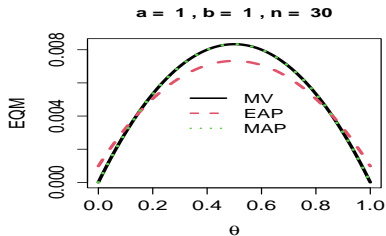
- $\mathcal{E}(\hat{\theta}_{\text{MAP}}) = \frac{n\theta + a - 1}{n + a + b - 2}.$

- $\mathcal{V}(\hat{\theta}_{\text{MAP}}) = \frac{n\theta(1 - \theta)}{(n + a + b - 2)^2}.$

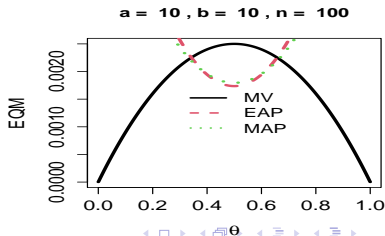
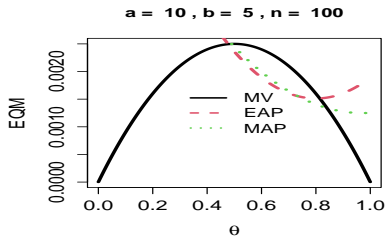
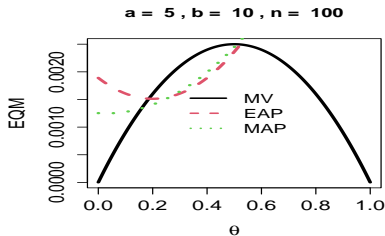
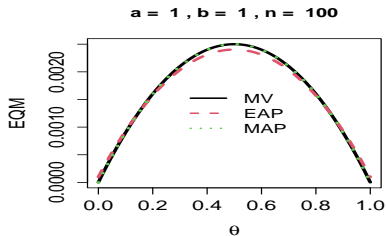
- $B(\hat{\theta}_{\text{MAP}}) = \frac{a - 1 - \theta(a + b - 2)}{n + a + b - 2}.$

- $\text{EQM}(\hat{\theta}_{\text{MAP}}) = \frac{n\theta(1 - \theta) + [a - 1 - \theta(a + b - 2)]^2}{(n + a + b - 2)^2}.$

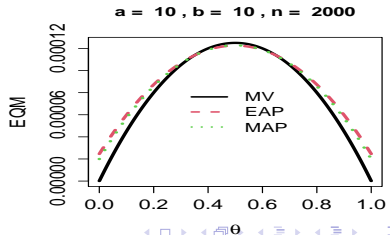
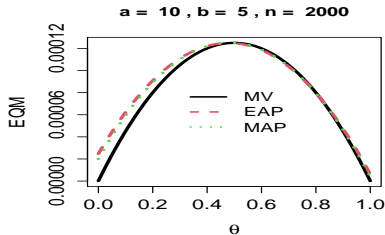
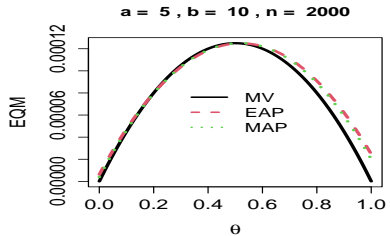
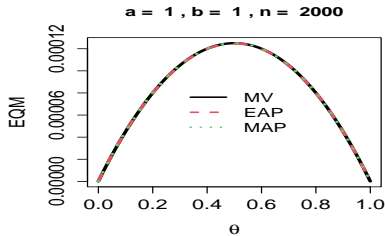
Comparação dos EQM's ($n = 30$), para diferentes (a,b)



Comparação dos EQM's ($n = 100$), para diferentes (a,b)

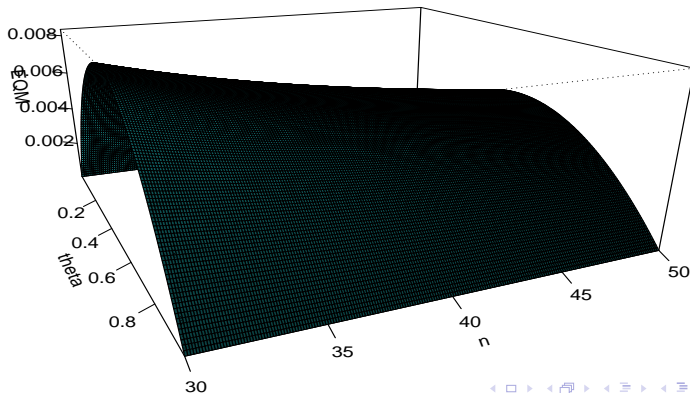


Comparação dos EQM's ($n = 2000$), para diferentes (a,b)



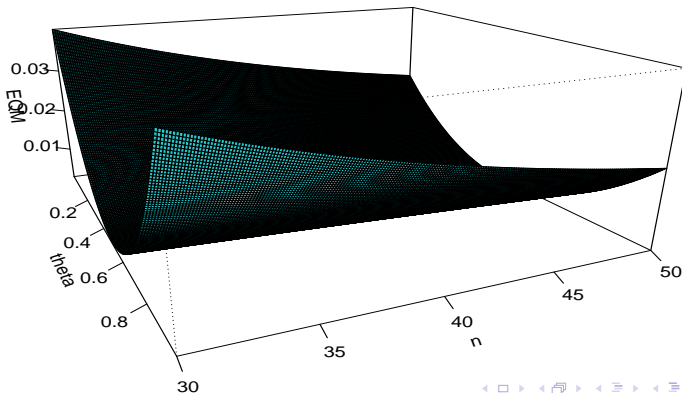
Comportamento do EQM em função de θ e n

Estimador de máxima verossimilhança



Comportamento do EQM em função de θ e n

Esperança à posteriori, $a = 10$, $b = 10$



Comportamento do EQM em função de θ e n

Moda à posteriori, $a = 10$, $b = 10$

