Biomatemática **25** *(2015), 185–200* ***ISSN 1679-365X***
Uma Publicação do Grupo de Biomatemática IMECC – UNICAMP

An approach via fuzzy sets theory for the dynamics of the soybean aphid and its predator

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Abstract. This paper uses a methodology based in fuzzy sets theory in order to describe the interaction between the prey, *Aphis glycines* (Hemiptera: Aphididae) - the soybean aphid, and its predator, *Orius insidiosus* (Hemiptera: Anthocoridae). Economic thresholds were already developed for this pest. The aim of this investigation was to develop a simple and specific methodology by fuzzy rule-based system to help enhance decision making tools for biological control of this pest. The model includes biotic (predator) and abiotic (temperature) factors, which affect the soybean aphid population dynamics. The paper also includes a comparison between the fuzzy model and data reported in the literature. This model is very useful to predict timing and releasing numbers of predators for soybean aphid biological control.

Palavras-chave: *Fuzzy sets; Fuzzy systems; Mamdani inference method; Prey; Predator; Soybean aphid.*

1. Introdution

The soybean aphid, *Aphis glycines* (Hemiptera: Aphididae), is an invasive herbivore new to North America. It was first discovered in North America

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in Wisconsin in late July 2000 infesting soybean crop. Natural enemies have been observed to attack this pest, but the economic threshold was developed for chemical control (Yoo e OŃeil, 2009).

This current work describes a method based in Fuzzy Sets Theory, introduced in the sixties by Zadeh (Zadeh, 1965), to build a model to evaluate the interaction of the prey, A. glycines (the soybean aphid) and the predator, *Orius insidiosus* (Hemiptera: Anthocoridae), and the effects of the temperature in the growth of the prey population.

In this case the dynamic model results in a fuzzy model that preserves the biological meaning and nature of the predator-prey model. The aim of this investigation is to propose a specific methodology to enhance current decision making tools to control this important pest. Although the soybean aphid is still a quarantine pest in Brazil, the predator is present. Therefore, before any eventual invasion, a predictive model to enhance biological control program is desirable.

Brazil is the second exporter of soybean at present, after the USA and before Argentina. According to the Bureau of Agriculture of the USA, it has been estimated that Brazil will be the largest soybean exporter in 2023. Therefore, considering the economic importance of soybean to Brazil, we need to be prepared for effective proposals to control soybean aphids or any other invasive pest.

2. Soybean aphid

The soybean aphid is a small sap-succking and partenogenetically reproducing insects. Females are produced during spring and summer where as males and females are produced during late fall when mated females lay diapausing eggs on buckthorn Rhamnus sp, an overwintering host. It is one of the major insect pests of soybean reproducing stages when high populations may build up and can cause great damage (Ragsdale et al., 2004).

Economic thresholds for the soybean aphid were developed and varied from 250 to 273 aphids/plant (Ragsdale et al., 2007). These thresholds are mainly for pesticide use, despite the fact that some naturally occurring predators such as the multicolored Asian lady beetle, Harmonia axyridis (Coleoptera: Coccinelidae) and the insidious flower bug, *Orius insidiosus* (Hemiptera: Anthocoridae) which is the commonest and the most important predator (Ragsdale et al., 2007). The insidious flower bug can significantly slow soybean aphid population growth, particularly during hot July weather (Brosius et al., 2007).

Soybean aphid populations can grow to extremely high levels under favorable environmental conditions. Reproduction and development is faster when temperatures are between 25°C and the mid 29°C when population can double in two or three days. The aphids are greatly affected by temperatures above 30°C, and they are reported to begin to die when temperatures reach 35°C (McCornack et al., 2004).

3. Predation

Predation is an example of the interaction between two populations that results in negative effects on growth and survival of one population (prey) and in positive or beneficial effects to the other (predator). A predator is an organism that hunts and kills other organisms for food (Edelstein–Keshet, 1998). Prey populations grow when predators are absent. Predators depend on the presence of their prey to survive. An encounter is assumed to decrease the prey population and increase the predator population by contributing to their growth.

We present hypotheses that characterize a predator-prey model, that is:

- if the number of predators is small, the number of prey increases;
- if the number of predators is large, the number of prey decreases;
- if the number of prey is large, the number of predators increases;
- if the number of prey is small, the number of predators decreases.

According to these hypotheses, we have established fuzzy rule base instead of usual differential equations that characterize the classic deterministic models. This rule base replaces differential equations, which characterize the classic deterministic models used to model the dynamics between prey and predators. These hypotheses are qualitative information that allow us to propose rules that relate (at least partially), the numbers in the populations with their own variations (Peixoto et al., 2008). In fact, our main interest in this paper is to elaborate a predator-prey model that represents the interaction between soybean aphids (prey) and their predators using fuzzy rule-based systems.

4 The Mathematical model

This work describes a methodology based in Fuzzy Sets Theory to elaborate a model that studies the interaction of the prey, *A. glycines* (soybean aphids), and the predator, *Orius insidiosus*, and the effects of the temperature in the growth of the prey population.

Next we develop brief reviews of the concept of fuzzy set and fuzzy rule-based system, and we detail the fuzzy model suggested in this paper.

4.1 Fuzzy rule-based system

Fuzzy sets and fuzzy logic have become one of the emerging areas in contemporary technologies of information processing. Fuzzy Sets Theory was first developed by Zadeh (Zadeh, 1965) in the mid-1960s to represent uncertain and imprecise knowledge. It provides an approximate but effective means of describing the behavior of the system that is too complex, ill defined, or not easily analyzed mathematically, and this is our case.

A fuzzy set A is characterized by a membership function, ψ_A, mapping the elements of a domain X to the unit interval $[0, 1]$. That is, $\psi_A : X \to [0, 1]$. Clearly, a fuzzy set is a generation of the concept of a set whose membership function takes on only two values $\{0, 1\}$, that is, the characteristic function of A, $\chi_A : X \to \{0, 1\}$.

Fuzzy variables are processed using a fuzzy rule-based system. A general fuzzy rule-based system consists of four components: an input processor (fuzzification), a fuzzy rule base; a fuzzy inference method and an output processor (defuzzification). These components process real-valued inputs in order to provide real-valued outputs.

The fuzzification is the process in which the input values of the system are converted into appropriate fuzzy sets of their respective universes. It is a mapping of the domain of real numbers to a fuzzy range (Klir e Yuan, 1995). Expert knowledge plays an important role in building the membership functions for each fuzzy set associated with the inputs.

The rule base characterizes the objectives and strategies used by specialists in the area through a linguistic rule set. It is composed of a collection of fuzzy conditional propositions in the form if-then rules (Klir e Yuan, 1995).

The fuzzy inference method performs an approximate reasoning using the compositional rule of inference. A particular form of fuzzy inference of interest here is Mamdani's method (Pedrycs e Gomide, 1998). In this case, it aggregates the rules through the logical operator OR, modeled by the maximum operator and, in each rule, the logical operators AND and THEN are modeled by the minimum operator (Pedrycs e Gomide, 1998). The logic of decision to be made, incorporates the structure of inference of the rule base and uses fuzzy implications to simulate the decisions (Klir e Yuan, 1995). It generates actions inferred from consequents a set of input conditions - antecedents.

Finally, in defuzzification, the value of the output linguistic variable inferred from the fuzzy rule is translated to a real value. The output processor's task is to provide real-valued outputs using defuzzification which is a process that chooses a real number that is representative of the inferred fuzzy set. A typical defuzzification scheme adopted in this paper is the centroid or center of mass method (Pedrycs e Gomide, 1998).

According to (Klir e Yuan, 1995), fuzzy set approaches have been developed for special purposes where the information base is vague and/or imprecise. Under these conditions, fuzzy techniques allow more accurate conclusions in comparison to the other approaches which cannot be applied successfully because of lack of data.

4.2 The formulation of mathematical model

The fuzzy model is composed by a predator-prey system connected to a prey-temperature system.

The variables of one system are number of preys - x, number of the predators - y (input variables) and their variations (output variables). The fuzzy sets of the input variables are small, small medium, large medium, large (that is, the triangular functions defined in the interval [0;1000] for the number of prey and in the interval [0;0.3] for the number of predators) and the fuzzy sets of the output variables are small positive, large positive, small negative, large negative (that is, the triangular functions defined in the interval [-0.4;0.4] for the variation of the preys - x' - and in the interval [-0.1;0.2] for the variation of the predators - y') .

The variables of the other system are temperature (input variable) and the variation of the number of preys (output variable).Now, the fuzzy sets of the input variable are low, moderate, high, very high (that is, the triangular functions defined in the interval [0;35] and the fuzzy sets of the output variable are small positive, positive, large positive, negative (that is, the triangular functions defined in the interval [-0.4;0.5] (McCornack et al., 2004).

Considering the hypotheses of the predation, we have elaborated 16 rules of the first system **(S1)**:

1. If (x is small) and (y is small) then ($x\prime$ is large-positive) and ($y\prime$ is large-negative)
2. If (x is small-medium) and (y is small) then ($x\prime$ is large-positive) and ($y\prime$ is small-negative)
3. If (x is large-medium) and (y is small) then ($x\prime$ is large-positive) and ($y\prime$ is small-positive)
4. If (x is large) and (y is small) then ($x\prime$ is large-positive) and ($y\prime$ is large-positive)
5. If (x is small) and (y is small-medium) then ($x\prime$ is small-positive) and ($y\prime$ is large-negative)
6. If (x is small-medium) and (y is small-medium) then ($x\prime$ is small-positive) and ($y\prime$ is small-negative)
7. If (x is large-medium) and (y is small-medium) then ($x\prime$ is small-positive) and ($y\prime$ is small-positive)
8. If (x is large) and (y is small-medium) then ($x\prime$ is small-positive) and ($y\prime$ is large-positive)
9. If (x is small) and (y is large-medium) then ($x\prime$ is small-negative) and ($y\prime$ is large-negative)
10. If (x is small-medium) and (y is large-medium) then ($x\prime$ is small-negative) and ($y\prime$ is small-negative)
11. If (x is large-medium) and (y is large-medium) then ($x\prime$ is small-negative) and ($y\prime$ is small-positive)
12. If (x is large) and (y is large-medium) then ($x\prime$ is small-negative) and ($y\prime$ is large-positive)
13. If (x is small) and (y is large) then ($x\prime$ is large-negative) and ($y\prime$ is large-negative)
14. If (x is small-medium) and (y is large) then ($x\prime$ is large-negative) and ($y\prime$ is small-negative)
15. If (x is large-medium) and (y is large) then ($x\prime$ is large-negative) and ($y\prime$ is small-positive)
16. If (x is large) and (y is large) then ($x\prime$ is large-negative) and ($y\prime$ is large-positive)

And considering the influence of the temperature in the growth of prey population, we have elaborated 4 rules of the second system **(S2)**:

1. If the temperature is low, then the variation of aphids is small positive
2. If the temperature is moderate, then the variation of aphids is positive
3. If the temperature is high, then the variation of aphids is large positive
4. If the temperature is very high, then the variation of aphids is negative.

Hence, we have obtained the variation rates of the populations from Mamdani Inference method and the fuzzification of the center-of-gravity.

In the numerical simulations performed we have observed the variation of the number of the prey and the number of the predators considering the temperature. In order to achieve this, we have considered an initial number of aphids, $x_0 = x(t_0)$, an initial number of predators, $y_0 = y(t_0)$, in a plant and the temperature, T_0 in the instant t_0.

From **(S2)** we have obtained Δx_0 (variation of x_0 due the temperature in t_0). The fuzzy predator-prey system **(S1)** produces $x\prime_0$ and $y\prime_0$ as output values from the initial conditions in order to get x_1 and y_1. For each iteration, we have the following:

$$\begin{aligned} x(t_{i+1}) &= x(t_i) + h * (x\prime(t_i) + \Delta x(t_i)) \\ y(t_{i+1}) &= y(t_i) + h * y\prime(t_i) \end{aligned} \tag{4.1}$$

that is, Euler's Method, where h is the increment.

5 Results

5.1 Computer simulations

We have obtained the variation of the number of the prey and the number of the predators in the numerical simulation. Let be $x_0 = x(t_0)$ an initial number of aphids and $y_0 = y(t_0)$ be an initial number of predators per plant (input variables of the system **(S1)**) and the time-at-temperature derived from Figure 1.

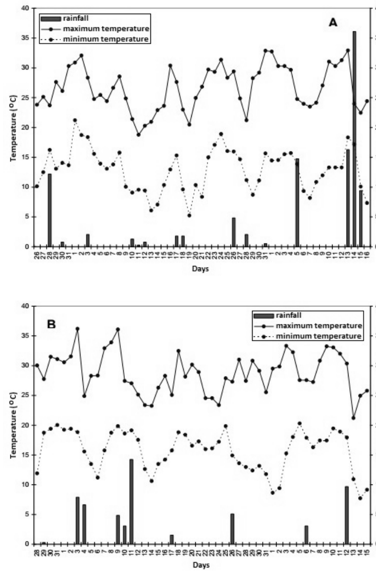

Figura 1: Climatic parameters (temperature and rain) registered during the soybean aphid infestation periods in 2004 (A) and 2005 (B). Data obtained from weather station at Haskell Agricultural Laboratory (Northeast Research and Extension Center), Concord, NE, USA.

Simulations of the trajectories produced by the fuzzy model follow the steps below:

- given an initial number of the prey population ($x(t_0)$) and an initial number of the predator population ($y(t_0)$) as input data of the one fuzzy rule-based system (**(S1)**;
- the fuzzy rule-based system of the predator-prey type gives the output data: $x\prime(t_0)$ and $y\prime(t_0)$;
- given an initial temperature T (mean temperatura in Figure 2), by **(S2)** we estimate Δx_0;
- from (1) we find $x(t_1)$ and $y(t_1)$;
- $x(t_1)$ and $y(t_1)$ are the input data of the fuzzy rule-based system of the predator-prey type (S1) and so forth.

The phase plane by this fuzzy system (dashed line) and the phase plane by the real data (Hunt, 2007) (black points) are illustrated in figures 2 and 3.

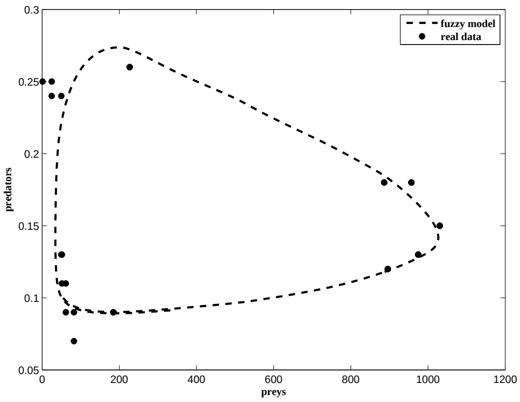

Figura 2: The phase plane by the fuzzy system and the phase plane by the real data with $x_0 = 24.46$ (initial number of aphids), $y_0 = 0.04$ (initial number of predators) and the temperature are available in Figure 1 in 2004.

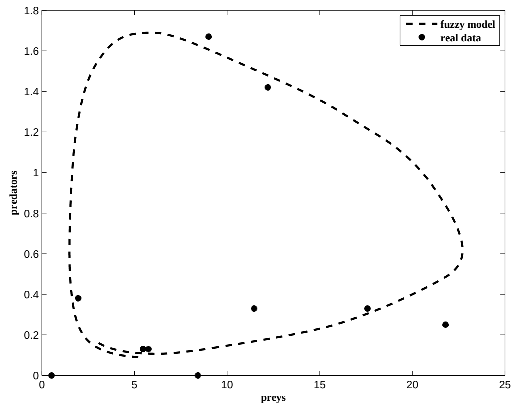

Figura 3: The phase plane by the fuzzy system and the phase plane by the real data with $x_0 = 1.96$ (initial number of aphids), $y_0 = 0.38$ (initial number of predators) and the temperature are available in Figure 1 in 2005.

5.2 Discussion

In section 4.2 we propose the fuzzy model to simulate soybean aphid population dynamics, that includes biotic (predator) and abiotic (temperature) factors. This model is based on fuzzy system **(S1)** that relate the input variables (number of prey, number of predators) with the output variables (variation of prey and variation of predators) and **(S2)** . The use of a fuzzy rule-based system instead of usual differential equations which characterize the classic deterministic models, because many parameters of the differential equations are not available. On the other hand, the qualitative information and data reported in the literature (Hunt, 2007) could contribute to the appropriate elaboration of the rule base and the fuzzy sets.

Section 5.1 demonstrates that the fuzzy mathematical model provides the phase plane that preserves the characteristics of the phase plane of a predator-prey model, that is, dynamic model results in a fuzzy model that preserves the biological meaning and nature of the predator-prey model.

As demonstrated by the simulations in Section 5.1, we can see clearly that the temperature influences the growth of the aphids population (McCornack et al., 2004) and a comparison between the fuzzy model and real data (Hunt, 2007).

We believe that this mathematical model will be very useful to predict the timing and the number of predators released for soybean aphid biological control. Although this pest is not currently present in South America, the model is important to simulate possible scenarios in a soybean plantation.

6 Conclusion

This study has suggested that the use of the fuzzy sets theory in Ecology may represent the interaction among species in the environment where the available data are few or qualitative. We have used intuitive hypotheses of the dynamics of aphids-insidious flower bug and data reported in the literature to elaborate the model without explicit differential equations. It was clear the influence of temperature on the growth of aphids population.

We have considered the Fuzzy Sets Theory as a great contribution to the construction of mathematical models, mainly when parameters of the differential equations are not available.

We would like to highlight the advantages of using fuzzy rule-based models compared to the deterministic models:

- The input and output sets of fuzzy rule-based systems may be easily defined by experts, that is, specialists who may know when the population of a particular species is small, large and so forth of the predators population over time.

- We have used a rule base instead of systems given by equations, eliminating the difficulty of obtaining the parameters.

- Several differential equations parameters of the predator-prey type systems are not available.

- If it is necessary to know the parameters, they may be obtained through a curve fitting of the solution generated by the fuzzy model. That is, the parameters may be obtained through a curve fitting procedure from the solutions taken from the fuzzy rule-based model, i.e., it is imposed that the solution curve created by the fuzzy rule-based model be one solution to the deterministic system.

The future work will develop a simple and specific methodology by fuzzy rule-based system to help enhance decision making tools for biological control of this pest.

Acknowledgments

The first author acknowledges the Coordination for the Improvement of Higher Education Personnel (CAPES) and the second author acknowledges the National Council for Scientific and Technological Development (CNPq), project numbers 305862/2013-8, for the financial support.

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