Biomatemática ***17*** *(2007), 1–8* ***ISSN 1679-365X***
Uma Publicação do Grupo de Biomatemática IMECC – UNICAMP

Considerations to continue researches concerning agricultural products drying process through numerical simulation

Mariangela Amendola[1],
FEAGRI – UNICAMP, 13.081-970, Campinas/SP, Brazil
Phone: (55) 19 35211065, Fax: (55) 19 35211004

Abstract This work discusses the arguments that lead to the implementation choice between two finite difference schemes to solve the differential equation that models a specific drying process of agricultural product. The main purpose of using the methodology of mathematical modeling and numerical simulation is to compare its theoretical results with experimental data, in the least square method sense, in order to be able to identify thermal properties of the product. There are considered experimental data obtained from the soybean drying process and the one-dimensional mathematical model based on Fourier's law, under initial and boundary conditions according to the conducted experiment. All computations were implemented with MATLAB 6.5. As the explicit scheme needs to be carried out satisfying the stability criteria, it is around 46 times slower than the implicit scheme for the same temporal mesh; 98 when the Crout Method is used; and 980 times when using Crout and a larger temporal mesh— allowed only for the implicit scheme. To continue drying researches using other products we recommend the implicit scheme, mainly due to the number of simulations required.

Keywords: *Implicit scheme; Explicit scheme; Thermal properties; Soybean.*

1. Introduction

Knowing the drying kinetics as well as the parameters and thermal properties that characterize the drying process of agricultural products has become important to support the application of technologies to assure these products conservation. This experimental research area has shown improvements all over the world, as can be found in specific literature for post-harvest technology.

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[1] amendola@agr.unicamp.br

Particularly at College of Agricultural Engineering at University of Campinas, located in Brazil and known as FEAGRI/UNICAMP, pioneer-research work has been carried out using also applied mathematical as an important tool by Ito et al. (2002). In this research they used one algorithm based on the explicit scheme of the finite differences method, which was implemented with MATLAB 6.1, to solve the differential equation based on Fourier's law according to the initial and boundary condition based on a specific experimental procedure performed previously.

Despite the obtained values of the thermal property of the agricultural product dried, the used scheme requires a complementary extern algorithm to assure the stability criteria, which was taken according to the same orientation that appear in Pirozzi e Amendola (2005).

This request, as it is known, leads to more computational effort than that spent when using the implicit scheme, which has been used to similar researches (e.g. Amendola e Teruel (2002); Amendola (2003); Amendola e Pirozzi (2004); Amendola (2004); Amendola e Teruel (2005); Amendola e Pirozzi (2005); Amendola e Sarria (2005); Amendola e Queiroz (2005); Amendola (2005)).

Thus, searching for methodologies to improve this research, following the same investigation performed to the precooling process (Amendola e Pirozzi (2005)), we discuss the arguments that lead to the implementation choice of these two kinds of finite difference scheme: explicit and implicit, and also to the adequate choice of some included algorithms.

2. Material and methods

The experimental data considered were selected from those recorded from the soybean drying experimental process as described in Ito et al. (2003).

To conduct this experiment a cylindrical equipment was constructed which allows the drying product be placed in such a way that the process occurs in the spatial domain defined between two cylinders with radii $R_1 = 0.013$ [m] and $R_2 = 0.049$ [m], subject to a heat source, $qf = 393.7$ [W/m^2], placed at its central axis.

For a certain height of the equipment and at four positions along this spatial domain the values of soybean temperature, T [°C], were recorded along the time, t [s], approximately at each 100s during 6000s.

The following fixed parameters of the process or product were considered: initial temperature $T_0 = 23.1$ [°C], density = 1180 [kg/m^3] and heat capacity $C_p = 1970$ [J/kg°C].

The conductivity thermal value, k [W/m°C], is the unknown of this investigation. For this product it can be considered as belonging to the interval $K = [0.1, 0.3]$, as seen in the literature.

To find the best k value using this theoretical methodology, different values for k must be used in the mathematical model that describes the heat transfer process, perform the simulation of the referred process and compare the corresponding results, in the least square sense, with the experimental data. This must be carried out varying these k values untill the best residual value is reached.

The mathematical model associated to this specific process of heat transfer is the same previously established in Ito et al. (2002), which is based on the equation of Fourier's second law, one-dimensional, in cylindrical coordinates, under initial and boundary conditions based on the referred experimental procedure:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right), \quad R_1 < r < R_2$$

$$T(r,0), T_0\ ; \quad r \in [R_1, R_2]$$

$$-k\frac{\partial T}{\partial r}(R_1, t) = qf$$

$$\frac{\partial T}{\partial r}(R_2, t) = 0$$

The numerical simulation must be carried out according to some method in order to approximate the temperature values, $T = T(r,t)$, at points of the spatial domain and along the time.

To use both finite difference schemes, the following convention will be adopted:

$$T_j^i = T(i\Delta t, j\Delta r), \quad \text{for } j = 1, \ldots N_x \text{ and } i = 1, \ldots N_t;$$

where:

Δr is the spatial mesh inside the product;

Δt is the temporal mesh of the process;

N_x is the number of points in the spatial mesh and

N_t is the number of integrations carried along the time.

The analysis of the spatial mesh size influence must be carried out, and to use the explicit scheme a known stability criteria must be observed.

The explicit and implicit schemes we have used are defined respectively by the following expressions:

$$T_j^i = f_0\left(1-\frac{\Delta r}{r(j)}\right)T_{j-1}^{i-1} + \left(f_0\left(-2+\frac{\Delta r}{r(j)}\right)+1\right)T_j^{i-1} + f_0 T_{j+1}^{i-1}$$

$$f_0\left(1-\frac{\Delta r}{r(j)}\right)T_{j-1}^{i} + \left(f_0\left(-2+\frac{\Delta r}{r(j)}\right)-1\right)T_j^{i} + f_0 T_{j+1}^{i} = -T_j^{i-1}$$

where:

$$f_0 = \text{Fourier number} = \frac{k}{\rho C_p}\left(\frac{\Delta t}{\Delta r^2}\right)[].$$

The initial condition is:

$$T_i^1 = T_0, \quad i = 1, \dots N_x,$$

where T_0 is the initial temperature [°C].

The boundary conditions are:

$$T_1^i = T_2^i + b, \quad i = 1, \dots N_t.$$

where:

$$b = \frac{qf \times \Delta r}{k}.$$

and

$$T_{N_x}^i = T_{N_x-1}^i, \quad i = 1, \dots N_t.$$

The algorithms were implemented with MATLAB 6.5.

3. Results and discussion

The spatial mesh size can be taken as having 65 points and the stability criteria was observed, both as established to the case of explicit scheme as presented in Ito et al. (2002).

In Figure 1 the residual values are shown as a function of the value obtained for k when using the explicit scheme for k varying in subintervals of $K = [0.1, 0.3]$ established according to Bossarino et al. (2005) to automatically find the thermal property only at the central position. This was the selected position just because it is where the fit results show the worst agreement in all the cases studied by Ito et al. (2003). In the present case the best value k is $k = 0.223$ [W/m$^{\text{o}}$C], which is associated to the residual value of 0.3480.

The experimental data and the numerical results obtained from both of these schemes for the same best k value are shown in Figure 2.

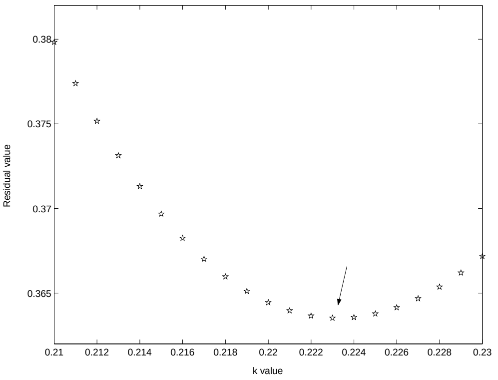

Figure 1: Residual values as a function of k values in $[0.21, 0.23]$.

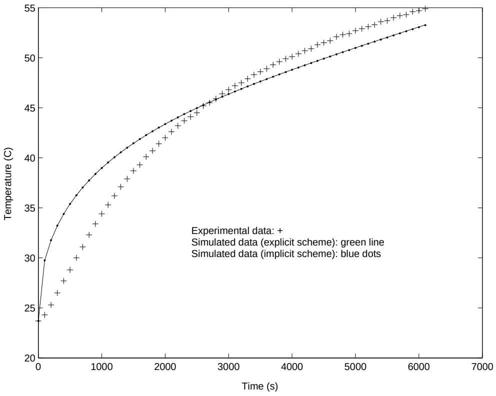

Figure 2: Experimental data and numerical results obtained using the explicit scheme and the implicit scheme, both to the best k value.

Despite the fact that the best value for k reached through the implicit scheme is, as expected, the same as that obtained through the explicit scheme, some considerations referring to the computational time must be reinforced.

Table 1 shows the computational time required for each numerical scheme according to the results in Amendola (2005). These results reveal disagreement with the observations pointed out in Ito et al. (2004) as regards to the computational effort, once the explicit scheme spent around 46 times the time spent by the implicit scheme for the same temporal mesh.

Furthermore Table 2 shows the computational time required for the implicit scheme implemented according to the classical literature, which means that the incorporation of improvements to the algorithm based on the implicit scheme, specially using the Crout Method to solve the linear system (Richtmyer e Morton (1967)) and allowing a coarse temporal mesh.

As the total investigation involves several simulations till the best value of the thermal property is reached at each point along the radii of the equipment, even knowing that the implicit scheme and Crout Method are avoided by most beginner researchers, due to its perceived complexity, the results presented in Table 2 show that this composed scheme is adequate to continue this kind of research.

Table 1: Kind of scheme and computational time

Scheme	Time(s)
Explicit	~ 98
Implicit	~ 2

Table 2: Implicit scheme and computational time

Implicit scheme	Time (s)
Crout method to same temporal mesh	~ 1
Crout method to a coarser temporal mesh	~ 0.1

4. Conclusion

The explicit scheme is around 46 times slower than the implicit scheme, for the same temporal mesh; 98 when as compared to the Crout method; and 980 times when compared to the Crout Method and a larger temporal mesh.

The numerical simulation methodology using the implicit scheme must be reinforced as the adequate methodology for the completion of this kind of investigation not only to this agricultural product and/or drying process.

Acknowledgements

The author would like to thank the scientific committee of the Eighth U.S. National Congress on Computational Mechanics (USNCCM8), Austin-TX, for the opportunity to discuss this research.

References

Amendola, M. (2003). Analysis of the parameters associated to numerical simulation of the heat transfer process in agricultural products. in "International Workshop on Information Technologies and Computing Techniques for the Agro-Food Sector", Barcelona.

Amendola, M. (2004). Minicurso: Sistemas de suporte à decisão para processos agrícolas: simulação numérica de processos de transferência de calor. páginas 112–138. in "Congresso Temático de Dinâmica e Controle da SBMAC", Ilha Solteira.

Amendola, M. (2005). Considerations to continue researches in drying process numerical simulation of agriculture products at feagri/unicamp. CD ROM. in "National Congress on Computational Mechanics", Austin.

Amendola, M. e Pirozzi, D. C. Z. (2004). Researches considerations in precooling and numerical simulation for agriculture products at feagri/unicamp. página 71. in "International Post Harvest Symposium", Verona.

Amendola, M. e Pirozzi, D. C. Z. (2005). Researches considerations in pre-cooling and numerical simulation for agricultural products at feagri/unicamp. *ISHS Acta Horticulturae*, 682:1599–1604.

Amendola, M. e Queiroz, M. R. (2005). Mathematical modeling and numerical simulation of the drying process to identify the mass diffusion coefficient of bananas. in "Iberian Latin-American congress on Computational methods in Engineering-CILAMCE", Guarapari.

Amendola, M. e Sarria, S. D. (2005). Mathematical modeling and numerical simulation of the forced air precooling process of fig fruit (ficus carica l.) roxo de valinhos. in “Iberian Latin-American Congress on Computational Methods in Engineering-CILAMCE”, Guarapari.

Amendola, M. e Teruel, B. J. (2002). Uso de un esquema implícito y de splines para la simulación numérica del proceso de enfriamiento de naranjas. página 141. in “Congreso Iberoamericano de Tecnología de Poscosecha y Agroexportaciones”, Chile.

Amendola, M. e Teruel, B. J. (2005). Esquema implícito e de splines para a simulação numérica do processo de resfriamento de frutas esféricas. *Revista Brasileira de Engenharia Agrícola e Ambiental*, 9:78–82.

Bossarino, D., Amendola, M., e Gaspar, C. R. (2005). Simulação computacional para a determinação de propriedades de grãos submetidos a um tipo de secagem. in “Congresso Nacional de Matemática Aplicada e Computacional”, São Paulo.

Ito, A. P., Amendola, M., e Park, K. J. (2002). Construção e avaliação de um sistema de determinação de condutividade e difusividade térmica em regime transiente. páginas 141–142. in “Congreso Iberoamericano de Tecnología de Poscosecha y Agroexportaciones”, Chile.

Ito, A. P., Amendola, M., e Park, K. J. (2003). Determinação da condutividade térmica de grãos de soja. *Revista Ibero-americana de Tecnología Poscosecha*, 5:82–85.

Ito, A. P., Park, K. J., e Ardito, T. H. (2004). Application of an explicit finite difference model to obtain thermal diffusivity of soybean. páginas 613–620. in “International Drying Symposium - IDS 2004”, São Paulo.

Pirozzi, D. C. Z. e Amendola, M. (2005). Modelagem matemática e simulação numérica do resfriamento rápido de morango com ar forçado. *Engenharia Agrícola*, 25:222–230.

Richtmyer, R. D. e Morton, K. W. (1967). *Difference methods for initial value problems*. Interscience, New York.