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Fuzzy fractional logistic model for cumulative cases of COVID-19

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Abstract. In this paper we deal with a fuzzy fractional logistic model to represent the dynamics of cumulative cases of COVID-19, using the fractional derivative of Caputo. The proposed model considers that the carrying capacity and growth rate parameters of the logistic differential equation are uncertain and given by fuzzy numbers. This assumption leads to a fuzzy initial value problem for which we obtain fuzzy solutions by means of the Zadeh extension principle. We use the proposed model to analyse the cumulative cases curve of five countries affected by COVID-19: China, France, Austria, Germany and South Korea.

Keywords: *Verhulst model, fractional differential equations, fuzzy numbers, fuzzy initial value problem, Zadeh extension principle, pandemic.*

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1. Introduction

The disease called COVID-19 by the World Health Organization is of unknown origin. The outbreak of this disease began in December 2019, causing respiratory infections. This disease is due to a new virus from the coronavirus family. There are three types of coronavirus known since the beginning of the millennium: SARS, MERS and COVID-19. The primary source of this virus is the bat, however another animal may be the intermediary for transmission to humans Guo et al. (2020). As pointed out in Zou et al. (2020), apparently there is a greater chance of transmission at the early stage of symptoms, which is when the viral ribonucleic acid (RNA) is at its highest level.

This article proposes to model the number of cumulative cases of COVID-19. This dynamics presents a logistic behavior, which considers growth limited by a carrying capacity, that is, similar to the classical logistic model given by a first order nonlinear ordinary differential equation. However, this simple model must consider uncertainties and memory effect, which can be done by an extension of the classical model through tools of fractional and fuzzy calculus.

Fractional differential equations (FDEs) are generalizations of the ordinary differential equations whose order is not necessarily a positive integer, that is, involve derivatives of arbitrary order. FDEs have attracted much attention due to their ability to model complex phenomena with memory and they have been widely used in engineering, physics, chemistry, biology, and other fields Zheng and Zhang (2017); Ibe (2013).

Fractional differential equation theory arise as a mathematical tool able to model systems with memory or even hysteresis. Hysteresis is a phenomenon associated with the memory effect on a system, when memory is related to the entire past period, and that usually occurs in epidemiological systems with behavioral effects on disease transmission. For example, the locomotion and habits of bats that contract rabies are altered, affecting the spread of this disease. What also generates this effect is the change in habits and behaviors of individuals due to control measures, such as social isolation, which have been applied to prevent and contain the spread of COVID-19. Thus, it is reasonable to consider the phenomenon of hysteresis in an epidemiological model of COVID-19 and, therefore, to use fractional differential equations Pimenov et al. (2012).

Moreover, classical models of differential equations do not consider the presence of uncertainties in the modelled phenomena. In order to incorporate uncertainties in the study of dynamic systems, many researchers have used fuzzy numbers to represent parameters and/or state variables of initial value problems (IVPs) Barros et al. (2017); Bede (2013). This approach leads to the class of problems called fuzzy initial value problems (FIVPs) Seikkala (1987). There are several methods to solve FIVPs, many of which generally deal only with first or second order (integer) differential equations, which are based, for example, on the Zadeh and sup-J extension principles De Barros and Santo Pedro (2017); Esmi et al. (2021); Mizukoshi et al. (2007) or on the notion of fuzzy derivatives Kaleva (1987); Bede and Gal (2005).

As already mentioned, COVID-19 is a new disease and since its emergence it has overloaded the health systems of several countries around the world (including developed ones). Thus, estimating the number of cases in a region is still a complex and important biological problem, since knowledge about the dynamics of this new disease is still preliminary, that is, there is a lot of uncertainty.

This paper proposes to combine the theories of fuzzy sets and FDEs to study the cumulative cases of COVID-19. Since we assume the phenomenon of hysteresis occurs in the spread of COVID-19 disease, we use fractional logistic system to model the cumulative cases of COVID-19 in a specific region or in a country. Moreover, we employ fuzzy numbers in order to deal with the uncertainty over the estimation of the carrying capacity and growth rate in the logistic equation. Finally, we employ the Zadeh extension principle to produce fuzzy solutions for the proposed fuzzy fractional logistic model.

This paper is organized as follows. In Section 2, we briefly review the classical logistic model, fractional calculus, some numerical methods for solving differential equations and some basic concepts and results from fuzzy set theory. In Section 3, we describe the proposed fuzzy fractional logistic model, the methodology to estimate the fuzzy parameters and establish a fuzzy solution via the Zadeh extension principle. Section 4 presents the fuzzy solutions for cumulative cases in five countries affected by COVID-19: China, France, Austria, Germany and South Korea. Finally, the final remarks and summary of the main contributions of this work is presented in Section 5.

2. Preliminary

This section introduces the main mathematical concepts used in this paper. First, we briefly review the classical logistic model. Then, the basic results of fractional calculus and numerical methods for solving differential equations are provided. Finally, some concepts from fuzzy set theory are also remembered.

2.1. Classical logistic model

The classical *logistic equation* is given by the following initial value problem Edelstein-Keshet (2005):

$$\begin{cases} x'(t) & = & rx(t)\left(1 - \dfrac{x(t)}{k}\right), \\ x(0) & = & x_0, \end{cases} \tag{1}$$

where $x(t)$ represents a biological population (for example, animals or viruses), r is the growth rate, k the carrying capacity, and $x(0)$ is an initial value for this population. In general, it is assumed that $r, k, x_0 \in \mathbb{R}^+$) Edelstein-Keshet (2005); Britton (2012). Moreover, in a classic logistic model, the carrying capacity (k) may be conceptualized as the maximum population that an ecosystem can sustainably support Edelstein-Keshet (2005); Britton (2012). In our case, k refers to the maximum number of individuals infected with COVID-19 that a given country can reach.

In this work, instead of using the first order logistic differential equation given in (1), we consider a fractional order logistic differential equation to include the hysteresis phenomenon in the proposed model for the spread of COVID-19.

2.2. Fractional Calculus

This subsection presents some concepts of fractional calculus.

Let $(L[a,b],\mathbb{R})$ be the set of all Lebesgue integrate functions from $[a,b]$ to $\mathbb{R}$ and let $(AC[a,b],\mathbb{R})$ be the set of all absolutely continuous functions from $[a,b]$ to $\mathbb{R}$.

Definition 1. Podlubny (1998) *The Riemann-Liouville fractional integral of* *order* $\gamma \in (0,1]$ *for* $f \in (L[a,b],\mathbb{R})$ *is given by*

$$(I^{\gamma}_{a^+}f)(t) = \frac{1}{\Gamma(\gamma)}\int_a^t (t-s)^{\gamma-1} f(s)ds, \ \ \textit{for } t > a,$$

where $\Gamma(\gamma)$ *stands for the gamma function.*

Note that if $\gamma = 1$, we have $(I^1_{a^+}f)(t) = \int_a^t f(s)ds$. Next, we obtain the Riemann-Liouville derivative in terms of the fractional integral of Riemann-Liouville.

Definition 2. Podlubny (1998) *The Riemann-Liouville derivative of order* $\gamma \in (0,1]$ *is given by*

$$({}^{RL}D^{\gamma}_{a^+}f)(t) = \frac{d}{dt}I^{1-\gamma}_{a^+}f(t) = \frac{1}{\Gamma(1-\gamma)}\frac{d}{dt}\int_a^t (t-s)^{-\gamma} f(s)ds, \ \ \textit{for } t > a.$$

Definition 3. Podlubny (1998) *Let* $f \in (L[a,b],\mathbb{R})$ *and suppose there exists* ${}^{RL}D^{\gamma}_{a^+}f$ *on* $[a,b]$. *The Caputo fractional derivative* ${}^{C}D^{\gamma}_{a^+}f$ *is given by*

$$({}^{C}D^{\gamma}_{a^+}f)(t) = \left({}^{RL}D^{\gamma}_{a^+}[f(\cdot) - f(a)]\right)(t), \ \textit{for } t \in (a,b].$$

Besides that, Podlubny Podlubny (1998) showed that if $f \in AC([a,b],\mathbb{R})$, then

$$({}^{\mathrm{C}}D^{\gamma}_{a^+}f)(t) = \frac{1}{\Gamma(1-\gamma)}\int_a^t (t-s)^{-\gamma} f'(s)ds, \ \ \forall t \in (a,b] \tag{2}$$

and

$$({}^{\mathrm{RL}}D^{\gamma}_{a^+}f)(t) = ({}^{\mathrm{C}}D^{\gamma}_{a^+}f)(t) + \frac{(t-a)^{-\gamma}}{\Gamma(1-\gamma)}f(a), \ \ \forall t \in (a,b].$$

For convenience of notation, we use the symbol $D^{\gamma}x$ instead of ${}^{\mathrm{C}}D^{\gamma}_{0^+}x$ to refer to the Caputo fractional derivative given in (2) with $a = 0$.

In this work, we extend the first order logistic model given in (1) by considering the Caputo fractional derivative of order γ. Thus, we obtain the following *fractional-order logistic equation* given by

$$\begin{cases} D^{\gamma}x(t) &= rx(t)\left(1 - \dfrac{x(t)}{k}\right), \\ x(0) &= x_0, \end{cases} \tag{3}$$

where $r, k, x_0 \in \mathbb{R}^+$ and $\gamma \in (0,1]$ is the fractional order of the fractional differential equation. Note that, we obtain the classical logistic model (1) if $\gamma = 1$. Existence and uniqueness of solution for the initial value problem (3) are established in El-Sayed et al. (2007).

Parameter estimation and numerical solution methods for initial value problem of the form (3) are described in the next subsection.

2.3. Parameter estimation and numerical solution

In this paper, we use the least squares method to determine all the parameters (r, k, γ) of the fractional logistic model (3). Then, we employ the predictor-corrector PECE method of Adams-Bashforth-Moulton Diethelm and Freed (1998) to produce a numerical solution of (3).

In the logistic model, the carrying capacity is uncertain and its estimation is usually a difficult task. Factors such as culture and specific population behavior impact the estimation of the maximum number (carrying capacity) of COVID-19 cases in a country. Furthermore, uncertainties about the behavior and propagation of this new disease, together with underreporting and inaccurate data, also motivate the use of fuzzy set theory as a mathematical tool to model and manipulate uncertain parameters in the proposed model. Some basic concepts and results of fuzzy set theory are presented in the next subsection.

2.4. Fuzzy set theory

A fuzzy set A of a universe X is characterized by a function $\mu_A : X \to [0, 1]$ called membership function, where $\mu_A(x)$ represents the membership degree of x in A for all $x \in X$ Zadeh (1965). For convenience of notation, we use the symbol $A(x)$ instead of $\mu_A(x)$. The class of fuzzy subsets of X is denoted by $\mathcal{F}(X)$. Note that each classical subset of X can be uniquely identified with the fuzzy set whose membership function is given by its characteristic function Barros et al. (2017).

Let $A \in \mathcal{F}(X)$, where X is a topological space. The α-levels of A are defined by

$$[A]_\alpha = \begin{cases} \{x \in X : A(x) \geq \alpha\}, & 0 < \alpha \leq 1 \\ cl\{x \in X : A(x) > 0\}, & \alpha = 0 \end{cases},$$

where $cl(Y)$ stands for the closure of the set Y Bede (2013); Barros et al. (2017). A fuzzy set A is contained in another fuzzy set B, denoted by $A \subseteq B$, if $[A]_\alpha \subseteq [B]_\alpha$ for all $\alpha \in [0, 1]$.

A fuzzy set A of $\mathbb{R}$ is said to be a fuzzy number if all α-levels are bounded, closed and non-empty intervals. Every α-level of a fuzzy number A is denoted by $[A]_\alpha = [a_\alpha^-, a_\alpha^+]$. The class of fuzzy numbers, denoted by $\mathbb{R}_\mathcal{F}$, includes the set of real numbers as well as the set of bounded closed intervals of $\mathbb{R}$ Barros et al. (2017); Gomes et al. (2015). A triangular fuzzy number is another well-known example of a fuzzy number. Recall that a triangular fuzzy number is denoted by the triple $(a; b; c)$, with $a \leq b \leq c$, such that $[A]_\alpha = [a + \alpha(b - a), c - \alpha(c - b)]$, $\forall \alpha \in [0, 1]$.

The Zadeh extension principle is a mathematical method to extend classical functions to deal with fuzzy sets as argument inputs. In another words, the Zadeh extension principle extends classical functions to fuzzy functions.

Definition 4. Zadeh (1975) *(Zadeh extension principle) Let $f : X \to Z$. The Zadeh extension of the function f is the fuzzy function $\widehat{f} : \mathcal{F}(X) \to \mathcal{F}(Z)$ that associates every fuzzy set A of X to a fuzzy set of Z. The membership function of $\widehat{f}(A)$ is given by*

$$\widehat{f}(A)(z) = \begin{cases} \sup\limits_{f^{-1}(z)} A(x) & , \text{ if } f^{-1}(z) \neq \emptyset \\ 0 & , \text{ if } f^{-1}(z) = \emptyset \end{cases},$$

where $f^{-1}(z) = \{x \ : \ f(x) = z\}$.

The next theorem reveals that the Zadeh extension principle of f at A, where $A \in \mathbb{R}_{\mathcal{F}}$, can be determined by means of α-levels.

Theorem 1. Nguyen (1978); Barros et al. (1997) *Let X and Z be metric spaces, $f : X \longrightarrow Z$ be a continuous function and A a fuzzy set of X such that $[A]_\alpha$ is compact for all $\alpha \in [0, 1]$. For all $\alpha \in [0, 1]$, we have*

$$[\widehat{f}(A)]_\alpha = f([A]_\alpha).$$

The Zadeh extension principle can be used to solve several problems such as fuzzy differential equations. Mizukoshi *et al.* Mizukoshi et al. (2007) showed that a solution for this type of fuzzy differential equation is given by the Zadeh extension of the (analytical or numerical) solution of the associated classical differential equation.

3. Fuzzy fractional logistic model

In this section we present our approach to model cumulative cases of COVID-19. First, using the data from South Korea, we compare the numerical solutions of the classical and fractional logistic model to validate the use of the latter. Next, we describe the fuzzy fractional logistic model, whose parameters are given by fuzzy numbers and the derivative is in the Caputo sense. Finally, we describe how the fuzzy parameters are determined and present the model solution.

3.1. Fractional logistic model

Fractional differential equation models can be used to model several complex biological problems. For instance, FDE models can be used to describe systems with memory associated with epidemiological problems (like the spread of COVID-19).

In order to show the advantage of an FDE model to describe this epidemiological phenomenon, we present the numerical solutions of the IVPs given in (1) and (3) to describe the accumulated data of COVID-10 in South Korea, as we see in Figure 1. For this, we use the methods described in Subsection 2.3 with the data obtained in Worldometers (2022); World Health Organization (2022). A brief look at Figure 1 reveals that the best approximation for the spread of COVID-19 in South Korea is based on the fractional logistic model.

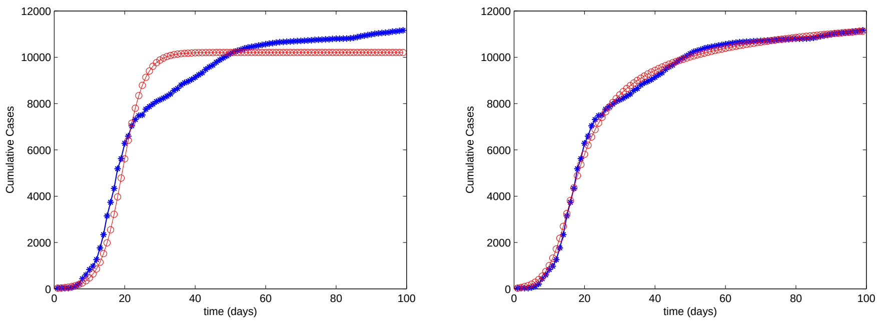

Figure 1: Numerical solutions of the classical and fractional logistic model in which the carrying capacity, growth rate and order of the fractional derivative are estimated using the least squares method to fit cumulative COVID-19 case data from South Korea. The left and right images exhibit the numerical solutions of (1) and (3), respectively. The red lines with circle markers represent the numerical solutions of the (classical and fractional) models and the blue lines with asterisk markers represent the current cumulative cases of COVID-19 (until May 23) in South Korea.

3.2. Fractional Logistic Model with Fuzzy Parameters

In general, the estimation of parameters and numerical solutions for PVIs can lead to errors in the description of a studied phenomenon. In addition, there are still many uncertainties about the behavior of COVID-19 and the availability of real data. Here, we represent the degree of such uncertainty by considering the parameters of the fractional logistic model (3) as fuzzy numbers. Therefore, we consider the following two fuzzy models.

First, we consider the carrying capacity as fuzzy number in (3), obtaining the following fuzzy initial value problem:

$$\begin{cases} D^{\gamma}x(t) &= rx(t)\left(1-\dfrac{x(t)}{K}\right) \\ x(0) &= x_0, \end{cases} \tag{4}$$

where $K \in \mathbb{R}_{\mathcal{F}}$, $r, x(0) \in \mathbb{R}^{+}$ and $x(t)$ represents the fuzzy solution for cumulative cases of COVID-19.

Second, we consider the growth rate as fuzzy number in (3). Thus, we obtain the fuzzy initial value problem:

$$\begin{cases} D^{\gamma}x(t) &= Rx(t)\left(1-\dfrac{x(t)}{k}\right) \\ x(a) &= x_0, \end{cases} \tag{5}$$

where $R \in \mathbb{R}_{\mathcal{F}}$, $k, x(0) \in \mathbb{R}^{+}$ and $x(t)$ represents the fuzzy solution for cumulative cases of COVID-19.

In the next subsection, we detail how the fuzzy parameters K and R are estimated. In addition, we describe our approach to obtain a fuzzy solution for the proposed FIVPs (4) and (5).

3.3. Fuzzy solution

Here, all deterministic parameters (k, r and γ) are determined from the IVP (3), using the least squares method and taking the cumulative case data of COVID-19 in Worldometers (2022); World Health Organization (2022).

For the FIVP (4) we consider a fuzzy carrying capacity given by a triangular fuzzy number $K = (k_{\min}\,;\, k\,;\, k_{\max})$. To determine the values of $k_{\min}$ and $k_{\max}$ we first find the (day) time t^* where the inflection point of the logistics curve occurs, that is, when the number of cumulative cases is approximately $\frac{k}{2}$. Then, $\frac{k_{\min}}{2}$ and $\frac{k_{\max}}{2}$ are equal to the number of cases in time t^*-1 and t^*+1 respectively.

For the FIVP (5) we consider the growth rate R as a fuzzy parameter given by a triangular fuzzy number $R = (r_{\min}\,;\, r\,;\, r_{\max})$, where $r_{\min} = r - 0.15r$ and $r_{\max} = r + 0.15r$.

Let U, V be open subsets of $\mathbb{R}$ such that $[K]_0 \subset U$ and $[R]_0 \subset V$ and let $x(\cdot, k, r, x_0)$ be the numerical solution of (3) obtained using the predictor-corrector PECE method with parameters k, r, and x_0. For every t, we define the operators $S_t(k) = y(t, k, r, x_0)$ for all $k \in U$ and $T_t(r) = y(t, k, r, x_0)$ for all $r \in V$. The fuzzy solution of (4) is given, for every t, by the Zadeh extension of S_t at $K \in \mathbb{R}_{\mathcal{F}}$ Mizukoshi et al. (2007); Esmi et al. (2021). Similarly, for every t, the fuzzy solution of (5) is given by the Zadeh extension of T_t at $R \in \mathbb{R}_{\mathcal{F}}$.

Since the parameter K (or R) is a fuzzy number, we can use the Zadeh extension principle Barros et al. (1997) to obtain a fuzzy solution $\widetilde{y} : [0, T] \to \mathbb{R}_{\mathcal{F}}$ for (4) (or (5)) given by $\widetilde{y}(t) = \widehat{S}_t(K)$ (or $\widetilde{y}(t) = \widehat{S}_t(K)$) for all $t \in [0, T]$ Mizukoshi et al. (2007); Esmi et al. (2021).

In the next section, we illustrate our approach through an application of the models (4) and (5) to describe the dynamics of COVID-19.

4. Results and discussion

In this section, we illustrate (see Figures 2-6) our approach by testing fractional fuzzy models (4) and (5) to describe cumulative cases of COVID-19 in five countries: China, France, Austria, Germany and South Korea. The logistic behavior is verified in the curve of cases accumulated in the respective periods related to each "wave" (of active cases). Therefore, we chose only the first "wave" of each country to verify our models, since the same can be done with data from the other "waves". In addition, China does not present more than one "wave". Furthermore, in all countries the second "wave" took longer to start than the following "waves", allowing a better visualization of stability in carrying capacity.

In all figures, the left and right images show respectively the fuzzy solutions of (5) and (4). The approximate fuzzy solutions are given in terms of the Zadeh extension of the numerical solution, as described in the previous section, and are represented in each figure through different shades of gray. Dashed red lines represent the 0-level and the gray-scale lines represent the endpoints of the α-levels for α varying from 0 to 1, where 0 and 1 correspond respectively to the lightest and darkest tones. Finally, Table 1 presents all estimated parameters.

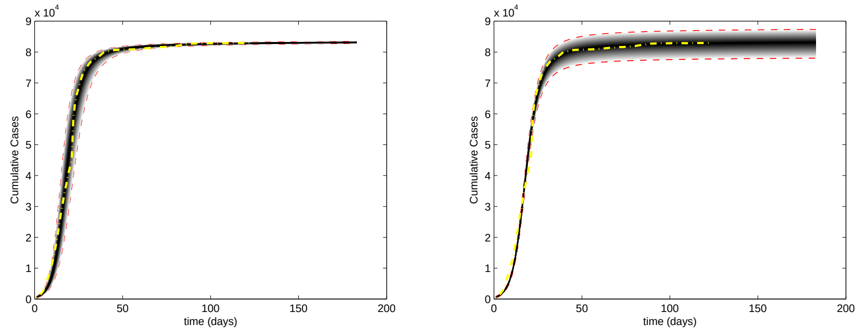

Figure 2: Fuzzy solutions of (4) and (5) for cumulative cases of COVID-19 in China. The dash-dotted yellow lines represent the curve of cumulative cases of COVID-19 until May 23, 2020.

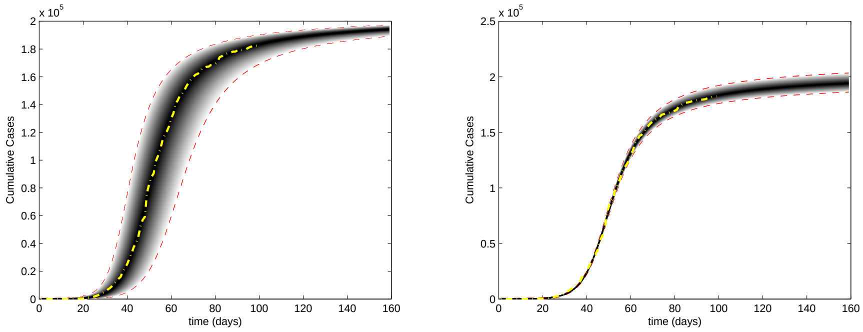

Figure 3: Fuzzy solutions of (4) and (5) for cumulative cases of COVID-19 in France. The dash-dotted yellow lines represent the curve of cumulative cases of COVID-19 until May 23, 2020.

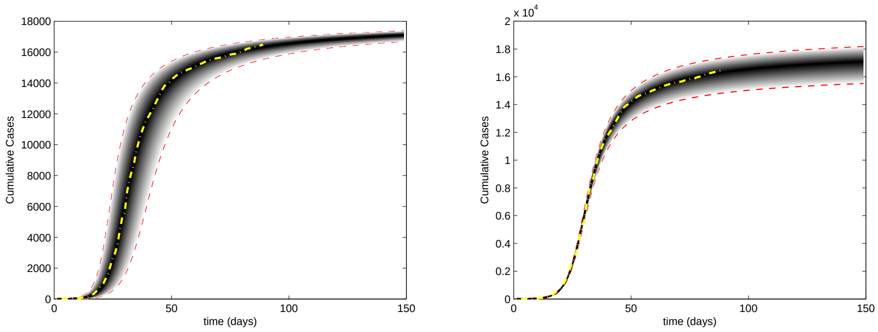

Figure 4: Fuzzy solutions of (4) and (5) for cumulative cases of COVID-19 in Austria. The dash-dotted yellow lines represent the curve of cumulative cases of COVID-19 until May 23, 2020.

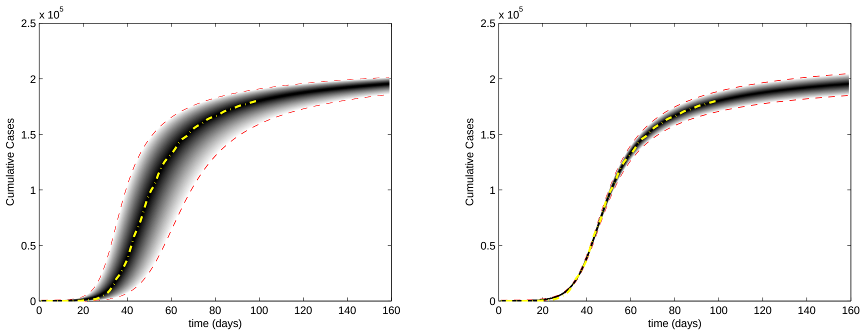

Figure 5: Fuzzy solutions of (4) and (5) for cumulative cases of COVID-19 in Germany. The dash-dotted yellow lines represent the curve of cumulative cases of COVID-19 until May 23, 2020.

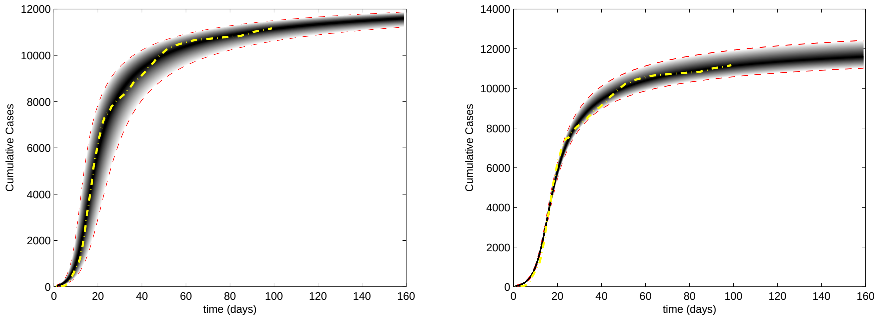

Figure 6: Fuzzy solutions of (4) and (5) for cumulative cases of COVID-19 in South Korea. The dash-dotted yellow lines represent the curve of cumulative cases of COVID-19 until May 23, 2020.

	k	r	γ
China	83687	0.3547	0.8439
France	207107	0.3451	0.6487
Austria	18454	0.5151	0.5603
Germany	222942	0.4406	0.5164
South Korea	13145	0.6615	0.4382

Table 1: Approximate parameters for each country

We can see that most of the actual data from cumulative cases of COVID-19 is contained in the fuzzy solution. Thereby, the models (4) and (5) present good approximations for the growth rate r and support capacity k of the analyzed countries, as can be seen in Table 1. The models also allow estimating the time (in days) in which the disease stabilizes in each country, reaching carrying capacity.

Moreover, it is worth noting that the parameter γ (derivative order) is associated with the speed of growth of the cumulative case curve, as well as the degree of memory effect involved in the dynamics Barros et al. (2021). Thus, from Figures 2-6 and Table 1 we can conclude that the country with the lowest memory effect and the fastest growing number of cases is China, followed by France, then Austria, Germany and South Korea. In fact, China has reached a carrying capacity of around $t = 50$ days, while in South Korea this time is over 150 days. It is worth noting that the slowest growing curve is the so-called "flattened curve". It also makes sense that in China there was the least memory effect in the first "wave", given that the pandemic began there, at a time when less was known about COVID-19.

5. Conclusions

The main contribution of this article is to propose a fuzzy fractional logistic model that represents well the dynamics of cumulative cases of COVID-19, which is currently one of the most important global problems.

This model uses a Caputo derivative and the parameters are fuzzy numbers, which allows to involve memory effect and uncertainties in the dynamics. We describe a brief methodology to determine the parameters of the proposed model with respect to COVID-19. Uncertainty is considered in the growth rate and also in the carrying capacity of the disease. Then, we find the numerical solution of the model, which is given in terms of the Zadeh extension principle.

We illustrate our approach using the proposed model to model cumulative cases of COVID-19 from 5 different countries: China, France, Austria, Germany and South Korea. The fuzzy solution was able to encompass most of the real data from all countries, as we can see in Figures 2-6. Furthermore, through the arbitrary order of the derivative it was possible to compare the degree of the memory effect and the speed of growth of the curve of cumulative cases, showing that in China the memory effect was smaller and the curve grew faster. Then France, Austria, Germany and South Korea, being the last countries with the greatest memory effect and where the so-called "flattened curve" occurred in greater intensity.

Finally, we emphasize that this work uses as an example only the first "wave", that is, the spread of the original variant of COVID-19. However, for the other "waves" the model can be applied, as long as the data are properly separated in the respective periods of each "wave" of active cases. Therefore, future works can be guided in this line of research.

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