Switching behaviour and stability in predator-preys systems

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Abstract

Some polyphagous predators switch their preference to prey depending on the relative prey frequencies, to elucidate how the switching property can evolve under natural selection, dynamical behaviours of two-prey and one-predator systems are analyzed. “switching” in predators which attack several prey species potentially can stabilize the numbers in prey populations. In switching, the number of attacks upon a species is disproportionately large when the species is abundant relative to other prey, and disproportionately small when the species is relatively rare. by using the newton-raphson’s method. In this paper is proved that the system generally has a stable three-species-co-existing equilibrium state. by considering a modification in the intrinsic growth rate of the prey population, our results have shown that of all possible communities, a higher fraction will be stable when predator switching is included in the model.

Keywords: Mathematical modeling, Predator-prey systems, Switched systems, Stability

1. Introduction

Population interaction between predator and prey, an essential warp of the ecological web of nature, has long been one of the central subjects of study in animal population ecology. One may find a vast number of contributions so far accumulated on this subject [1]. The first is the theoretical type of study founded by Lotka (1925) [2], Volterra (1926) [3] and Nicholson and Bailey (1935) [4]. Here various mathematical models have been constructed in terms of simultaneous differential or difference equations to mimic the behaviour of interacting predator
where $c_{ij}$ represents the relative prey preference, which is generally a function of the prey densities, $x_i$ and $x_j$. Murdoch (1969) [7] defined the term “switching” to refer to the case where $c_{ij}$ is a monotonically increasing function of the relative prey density $x_i/x_j$.

The equation (1) establishes a “null case”, that is, the expected results in the absence of switching. Essentially this null case is a simple model of predation, here we adopt the switching function given by Tansky [11]:

$$k_1(x_1, x_2) = \frac{a_1}{1 + (x_2/x_1)^n}$$

$$k_2(x_1, x_2) = \frac{a_2}{1 + (x_1/x_2)^n}$$

where $n$ is called the intensity of switching, switching becomes sharper as $n$ increases.

We lead with a framework predator-prey interaction established by Lotka [2] and Volterra [3] in a set of simultaneous differential equations given by:

$$\dot{X} = F(X) - G(X, Y)$$

$$\dot{Y} = U(G(X, Y)) - V(Y)$$

where $X = [x_1, \ldots, x_N]'$ and $Y = [y_1, \ldots, y_N]'$ represent densities of prey and predator, respectively. The functions $F = [F_1, \ldots, F_N]$; $G = [G_1, \ldots, G_N]$; $U = [U_1, \ldots, U_N]$ and $V = [V_1, \ldots, V_N]$ here represent the rates of prey reproduction (intrinsic growth), prey death due to predation, predator reproduction, and predator death, respectively.

Pelletier [17] and Tansky [11] considered $f(x)$ as having a exponential growth, due to Malthus (1798) [20]. In order to become the model more realistic, we consider here $f(x)$ as having a logistic growth in the population:

$$f_i(x_i) = r_i x_i (1 - x_i/k_i)$$

where $r_i$ and $k_i$ are positive constants. The constant $k_i$ is the carrying capacity of the environment, which is usually determined by the available sustaining resources.

In this paper the model consists of two trophic levels consisting of two preys and one predator (like in [11]) with the density of $x$ given by $[x_1, x_2, x_3]'$, respectively. the prey species interact only with the predator species that prey them and do not interact with another prey species. thus, incorporating eqn (4) and the considerations above into the framework of eqn (3), we have the model:
\[
\begin{align*}
\dot{x}_1 &= \left( r_1 - \frac{r_1}{k_1} x_1 - A_1(x_1, x_2) x_1 \right) x_1 \\
\dot{x}_2 &= \left( r_2 - \frac{r_2}{k_2} x_2 - A_2(x_1, x_2) x_2 \right) x_2 \\
\dot{x}_3 &= \left( -r_3 + A_1(x_1, x_2) x_1 + A_2(x_1, x_2) x_2 \right) x_3
\end{align*}
\]

where \( r_3 \) is predator’s natural mortality rate.

The Lotka-Volterra classic models for two-preys and one-predator are when \( A_1(x_1, x_2) \) and \( A_2(x_1, x_2) \) are constants, say,

\[
\begin{align*}
A_1(x_1, x_2) &= a_1 \\
A_2(x_1, x_2) &= a_2
\end{align*}
\]

In this case we have a model without switching. In order to consider the switching effect of predation, \( A_1(x_1, x_2) \) and \( A_2(x_1, x_2) \) are replaced by suitable functions given in (2), where:

\[
\begin{align*}
A_1(x_1, x_2) &= \frac{a_1 x_1^n}{x_1^n + x_2^n}, \quad a_1 = \text{const.} \\
A_2(x_1, x_2) &= \frac{a_2 x_2^n}{x_1^n + x_2^n}, \quad a_2 = \text{const.}
\end{align*}
\]

which have a characteristic property of a switching mechanism.

The effect of switching on the population stability has been discussed on the base of observational facts and also theoretical treatments, by many authors [1, 7-17, 19].

3. Methods

In order to compare the methodology proposed in this paper, we first consider the model (eqn (5, 6)) without predator switching, the equations governing the model are:

\[
\begin{align*}
\dot{x}_1 &= \left( r_1 - \frac{r_1}{k_1} x_1 - a_1 x_3 \right) x_1 \\
\dot{x}_2 &= \left( r_2 - \frac{r_2}{k_2} x_2 - a_2 x_3 \right) x_2 \\
\dot{x}_3 &= \left( -r_3 + a_1 x_1 + a_2 x_2 \right) x_3
\end{align*}
\]

The intrinsic growth rate of the prey species \( i \) \((i = 1, 2)\), \( r_i \); the carrying capacity for the prey species \( i \) \((i = 1, 2)\), \( k_i \); the death rate of the predator, \( r_3 \), and the predator’s efficiency search coefficient in relation to the prey species \( i \) \((i = 1, 2)\), \( a_i \) are each constant in time. To determine the stability of eqn (7), we obtain the fixed point considering the Newton-Raphson’s method to estimate the non-trivial steady state, having as the initial point population as the solution
obtained by Runge-Kutta’s method. Let \( X^* = [x_1^*, x_2^*, x_3^*] \) this solution. If any of the \( x_i^*, i=1,2,3, \) are negative, new choices are made for the parameters until choices that result in positive values for \( x_i^*, i=1,2,3, \) are obtained. To investigate the stability of this steady state, we shall use the Routh-Hurwitz criteria (all eigenvalues have negative real parts). If any of the densities drift off to zero or ever-increasing values, the fixed point is unstable.

Now, considering the effect of predator switching in the model (eqn (5)), let us consider the simplest form of the function eqn (7) with \( n = J \), then the equations governing the model are:

\[
\begin{align*}
\dot{x}_1 &= \left( r_1 - \frac{r_p}{k_1} x_1 - \frac{a_1 x_1}{x_1 + x_2} x_3 \right) x_1 \\
\dot{x}_2 &= \left( r_2 - \frac{r_p}{k_2} x_2 - \frac{a_2 x_2}{x_1 + x_2} x_3 \right) x_2 \\
\dot{x}_3 &= \left( -r_3 + \frac{a_1 x_1^2 + a_2 x_2^3}{x_1 + x_2} \right) x_3 
\end{align*}
\]  

(8)

where the parameters are defined like as eqn (7).

The relative interaction terms must appear in both equations since when a predator preys more heavily on a prey species both the population growth rate of the predator and the death rate of the prey will be modified by the same factor. Since as the model without switching, as the one with switching is non-linear, we apply the same methods in eqn (8) to determine and investigate the stability of the steady state.

4. Theoretical results

Prey switching has a strongly stabilizing influence (fig 2). In fig (1) we plot the evolution of eqn (7). All of the abundances have been scaled to their equilibrium values. The evolution graphed in fig (1) is an example of the general result that for many sets of parameters \( r, k, \) and \( a, \) the steady state is unstable such that eventually one of the species becomes extinct. In fig (1b, c, d) we also plot the phase portrait and plane of eqn (7). We have an unstable steady state \((x_1^*, x_2^*)\) that evolve into a limit cycle trajectory (fig 1 (c)), with fig 1 (a) showing the temporal variation of the populations with time. As in fig 1 (a) as in fig 1(d) we can clearly see that the prey species \( x_2 \) goes to extinction.

In fig (2) we plot the evolution of eqn (8) with the same coefficients \( r, k, \) and \( a, \) and initial values as in fig 1. The initial perturbations damp out indicating that the model with predator switching has a stable three-species co-existing equilibrium state for these parameter
values. We can confirm this result with the phase portrait and plane in fig 1(b, c, d) that exhibit stable spiral trajectories. Note the different scales of fig 1 (a) and fig 2 (a). Fig 1 (a) is plotted only up to \( t = 20 \) when one of the species becomes extinct. Fig 2 (a) is plotted for a longer period of time to indicate that the model is stable out to times much longer than the time at which species in fig 1 (a) become extinct. Also the vertical scale of fig 1 (a) is larger than fig 2 (a) indicating is greater variability.

Fig 1. Plot of the predator and prey population densities of the model eqn (7) as a function of time without predator switching. (a) temporal variation of the populations \( x_i, i = 1,2,3 \), with time; (b) phase portrait; (c) projection to the \((x_1, x_2)\) plane; (d) projection to the \((x_2, x_3)\) plane. The evolution of eqn (7) indicates an unstable fixed point.
5. Discussion

Using a generalization of Lotka-Volterra classic models, adding a special term to characterize it as switching predator model and comparing this with the other without switching we study the behaviour of the one predator two prey's population.

In previous works [21, 22], was analyzed mathematically the effect of changing the functional response in the predator-prey Rosenzweig-MacArthur type model, and in [23, 24] we study the local stability of switching systems by using two different approaches.

The theoretical results we obtained in this research are the beginning of a new approach we are working: switching predator on population. By comparing the established model we study the behaviour of the population by adding a switching term. We observe, for the model without switching and for different set of parameters considered that always one part of the population (the prey) went to extinction. The behaviour of the population changes its phase portrait, it left
its limit cycle for a stable spiral trajectory when switching is added converging asymptotically to its steady state.

Dynamical behaviour of prey predators system with predator switching can stabilize the system as a whole and ensure permanent co-existence. Ecologically speaking, permanence of a system means that all the populations are present and none of them will go to extinction.

We intend to follow this research by considering a general approach with wherever set of parameters, finding the theoretical fixed point and analyzing the stability of the population proposed with another method, like the Lyapunov method. Also, an alternative to study local stability in populations with switching is looking for models of the hybrid type $\dot{x} = f_i(x)$, where $x$ and $i$ are, respectively, the continuous and discrete variable for this kind of model and each of the $f_i$ are the law for the behaviour of the population. For this, is necessary an interdisciplinary interactions between biology and mathematics to do the modeling. Our efforts will be directed to get this objective.

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