# Numerical simulations of an oil spill accident in Guanabara Bay Rio de Janeiro, Brazil

R. F. Cantão[1], R. F. De Oliveira[2] & J. F. C. A. Meyer[1]
[1]*Department of Applied Mathematics, State University at Campinas, Brazil*
[2]*Department of Mathematics, Federal Rural University at Rio de Janeiro, Brazil*

# Numerical simulations of an oil spill accident in Guanabara Bay Rio de Janeiro, Brazil

R. F. Cantão[1], R. F. De Oliveira[2] & J. F. C. A. Meyer[1]
[1]*Department of Applied Mathematics, State University at Campinas, Brazil*
[2]*Department of Mathematics, Federal Rural University at Rio de Janeiro, Brazil*

## Abstract

In this work we present a classical evolution equation for the movement of oil slicks in marine water in its second phase (Fay, 1969), and, setting it in its variational formulation, discretize it with a view towards the use of the Finite Element Method (using first degree approximations for oil concentration). For simulating marine currents special upwinding techniques are adopted so as to eliminate main oscillations caused by numerical options. These currents are given by the solution of Stokes' equation using second order finite elements, and boundary conditions are obtained from data furnished by the Brazilian National Hydrographic Directory. Resulting currents were then used for a qualitative simulation so as to verify the model comparing it to information obtained after the accident and during clean-up operations. The model is also used to show the presence of oil in an environmentally protected area (called Guapimirim), an area of mangroves, essential for marine life in the bay. Program results are compared to circulated news on the oil spill movements and effects.

## 1 Introduction

In the early hours of January the 18$^{th}$, 2000, something like 1292 tons of oil leaked for four hours from the Brazilian national Petroleum Company (Petrobrás) facilities inside Guanabara Bay. This accident was due to a leak in a broken underwater pipeline, located quite far from the marine entrance to the Bay, where circulation caused by currents is very much smaller than wind-dominated movements. The oil slick initially moved because of currents induced by a wind from southwest, touching the shoreline along the northern tip of the bay, and subsequently moved down driven by a current induced by wind from the northwest, reaching the isle of Paquetá, as well as some beaches on the western shore of the Governador Island. This oil slick has severely affected the fishing communities in this region (mainly from the town of Magé) as well as the preservation area of wetlands inside Guanabara Bay. Media coverage of clean-up activities brought the oil spill into main news, and drew the attention of many sectors in society to the need of severe law-enforcement in this area. Press releases did not coincide completely with scientific fact.

In order to test both mathematical modelling techniques as well as numerical schemes for the simulation of oil spill movements, the approximate data relative to this accident were used in order to be able to compare simulation results with those obtained by media information, satellite images and local photographs.

## 2 Mathematical Model

Several authors have worked with oil spill movements using mathematical models based upon the classical Partial Differential Equation of the Conservation of Mass ([1], [3], [4], [7] and [8]). These models take into account the second phase of an oil spill, as defined by Fay [5] the so-called diffusive-advective phase, which describes the behaviour of an oil spill a few hours (4-6) after it occurs until some days (something between 10 and 15 days). Diffusion was considered as being constant, and a single term was used as a linear approximation to several processes of degradation. Coherent boundary conditions, for this level of approximation, were chosen, much in the line of Meyer, Cantão and Poffo (1998). The studied domain, identified as $\Omega$, is Guanabara Bay, discretized with the use of convenient software. Its boundary is given by $\partial\Omega$, the outer normal unit vector of which is given by $\eta$. The chosen model is therefore given in its classical formulation by:

Obtain $u = u(x,y;t)$ for $(x,y)\in\Omega\subset\mathbf{R}^2$, $t\in(0,T] \subset \mathbf{R}$, such that

$$\frac{\partial u}{\partial t} + \operatorname{div}(-\alpha\nabla u) + \operatorname{div}(\mathbf{V}u) + \sigma u = f, \quad \text{for}$$

$$u(x,y;0) = u_o(x,y), \quad (x,y)\in\Omega\subset\mathbf{R}^2, \text{ with} \tag{1}$$

$$u, f : \Omega\times[0,T]\to\mathbf{R}, \ \mathbf{V}:\Omega\to\mathbf{R}^2 \text{ and } \sigma:[0,T]\to\mathbf{R}.$$

Boundary conditions used for this case study use the fact that very little oil that washed up on beach shores was retained upon those shores (this part of the boundary will be identified as $\Gamma_1$), and that no oil reached the southern part of the bay (this part will be named $\Gamma_0$), so that the adopted boundary conditions will be, for $\partial\Omega = \Gamma_0 \cap \Gamma_1$ of mixed type, Dirichlet and von Neumann homogeneous:

$$\left.\partial u \middle/ \partial \eta\right|_{\Gamma_1} = 0 \text{ and } u|_{\Gamma_0} = 0. \tag{2}$$

For constant diffusivity, an approach which is both practical and coherent for Fay's second phase of an oil spill, this classical formulation becomes:

$$\frac{\partial u}{\partial t} - \alpha\Delta u + \operatorname{div}(\mathbf{V}u) + \sigma u = f, \tag{1'}$$

and we can now make the option for a variational formulation, seeking to use whatever benefits we may from weaker conditions imposed upon parameters, solutions and information along the boundaries. We will now consider the space for test functions as

$$H = \{ v \in H^1(\Omega) : v|_{\Gamma_0} = 0 \} \subset H^1(\Omega), \tag{3}$$

as well as the space in which we, theoretically at least, seek to obtain the solution u(x,y;t) as:

$$W = \{ v \in L^2((0,T], H^1(\Omega)) : \forall t \in (0,T], \frac{\partial v}{\partial t} \in L^2(\Omega) \text{ and } v|_{\Gamma_0} = 0 \}. \tag{4}$$

Formulation (1) becomes the following equation (in which we have used Green's Theorem):

$$(\frac{\partial u}{\partial t} | v) + \alpha(\nabla u \| \nabla v) + (V \bullet \nabla u | v) + \sigma(u | v) = (f | v), \forall v \in H. \tag{5}$$

The boundary conditions have been made part of this formulation, and the only condition that was not previously mentioned was that the velocity field is conservative, that is, div(V) = 0. This expression permits the choice of an approximation via Galerkin's Method by choosing a subspace $V_h$ of H, which is finite-dimensional and generated by chosen finite elements $\varphi_j$. This choice separates space and time variables, by the use of

$$u_h = \sum_{j=1}^{n} c_j(t)\varphi_j(x,y), \quad \forall \varphi_j \text{ from the basis } \beta \text{ of } V_h. \tag{6}$$

This will transform equation (5) into a system of Ordinary Differential Equations.

The initial condition, for this kind of second-phase model is given by the satellite image presented in daily papers (*Jornal o Globo,* January 23rd, 2000) 12 hours after the period in which the leak occurred. The resulting oil slick reached from well beyond western boundaries of the refinery region, touched the Mauá and Anil beaches of Magé county and was (at that time) limited on the eastern side by the point on the continent immediately north of Paquetá Island.

# 3 Approximate Solutions

Besides the choice for (6), we will also make an option for a second-order approximation method in time: Crank-Nicolson, and we will, therefore, define

$$u_h(x_j, y_j; t_n) \cong \sum_{j=1}^{n} c_j(t_n)\varphi_j(x, y) \cong \sum c_j^{(n)}.\varphi_j(x, y), \qquad \varphi_j \in \beta \subset \mathbf{V}_h. \quad (7)$$

The use of this expression in the weak formulation given by (4) besides the choice for $c^{(0)}{}_j = u_0(x_j, y_j)$, leads us to a linear system of equations in the unknowns $c_1{}^{(n+1)}, c_2{}^{(n+1)}, c_3{}^{(n+1)}, \ldots, c_N{}^{(n+1)}$:

$$\begin{aligned}
&\sum_{j=1}^{n} c_j^{(n+1)}\left[\left(1+\sigma\frac{\Delta t}{2}\right)(\varphi_j|\varphi_i)_\Omega + \alpha\frac{\Delta t}{2}(\nabla\varphi_j \,\|\, \nabla\varphi_i)_\Omega + \right.\\
&\qquad \left. + \frac{\Delta t}{2}\left(V_x\frac{\partial\varphi_j}{\partial x}|\varphi_i\right)_\Omega + \frac{\Delta t}{2}\left(V_y\frac{\partial\varphi_j}{\partial y}|\varphi_i\right)_\Omega\right] = \\
&= \sum_{j=1}^{n} c_j^{(n)}\left[\left(1-\sigma\frac{\Delta t}{2}\right)(\varphi_j|\varphi_i)_\Omega - \alpha\frac{\Delta t}{2}(\nabla\varphi_j \,\|\, \nabla\varphi_i)_\Omega - \right.\\
&\qquad \left. - \frac{\Delta t}{2}\left(V_x\frac{\partial\varphi_j}{\partial x}|\varphi_i\right)_\Omega - \frac{\Delta t}{2}\left(V_y\frac{\partial\varphi_j}{\partial y}|\varphi_i\right)_\Omega\right] + \Delta t(f^{(n+\frac{1}{2})}|\varphi_i)_\Omega
\end{aligned} \quad (8)$$

with $i = 1, \ldots, N$ and for given $c^{(0)} = (c^{(0)}{}_1, c^{(0)}{}_2, \ldots, c^{(0)}{}_N)$.

In order to obtain values for $V_x$ and of $V_y$ on each element, the numerical solution of Stoke's equation using second-order finite elements is used on the Guanabara Bay discretized domain, and the obtained results are added to wind-induced currents.

$$\begin{aligned}
-\operatorname{div}(\nabla\mathbf{V}) + \nabla p = \mathbf{f}, &\ \text{em } \Omega \subset \mathbf{R}^2 \\
\operatorname{div}(\mathbf{V}) = 0, &\ \text{em } \Omega \\
\mathbf{V} = \mathbf{V}_0, &\ \text{em } \partial\Omega
\end{aligned} \quad (9)$$

The boundary conditions for Stoke's equation were determined using data provided by the National Hydrographic Directory (DHN), a service provided by the Brazilian Navy. The time series of measures for surface currents were treated with the Matlab "Rose" function to obtain set patterns of behaviours. These were tested against historical data before adding wind terms. The obtained velocity fields are given in figures 1 (without wind) and 2 (velocity field).

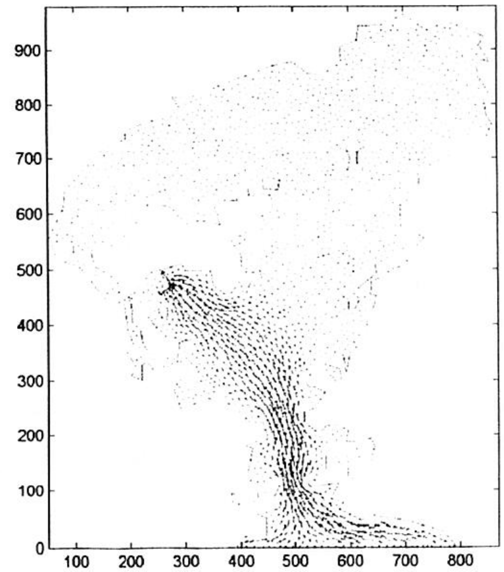


Figure 1 – Calculated velocity field (no wind influence).

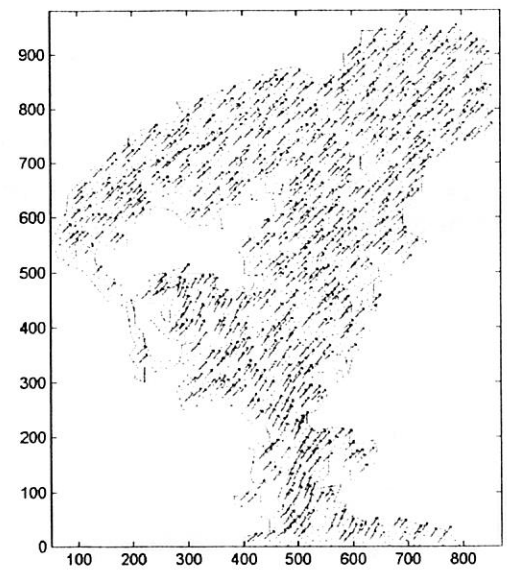


Figure 2 – Calculated velocity field (with wind influence)

With the output of this numerical approximation as input for the main program, and making use of upwinding techniques in order to eliminate numerical oscillations which plague the numerical solution of this kind of system, we obtained the following results for the oil spill movements during the first period, in which wind came in from the Southwest, causing the oil slick to spread along the northern shores of the bay touching on lands of the Refinery (REDUC), reaching beaches of the Magé region, as well as beginning to enter the Guapimirim mangroves, as indicated in figure 3. The used upwinding, at this point consisted in adding a fixed term in the direction from which wind blew, and subtracting this same term from the finite element when it is placed downwind, introducing discontinuity in the finite elements along boundaries separating the triangle which make up the grid mesh which is used to discretize the domain.

Initial condition

After 60 time steps

After 120 time steps

After 240 time steps

Using another common weather condition, the northwest wind, and adapting the velocities field, we obtained the following scenario, also considering the same initial condition:

After 100 time steps

After 200 time steps

After 300 time steps

After 400 time steps

# Conclusions

The main improvement upon our previous work is the inclusion of the Stokes equations to model water circulation. Before that, the velocity fields were created in a hand-made manner, using only qualitative information (as in [7]).

Simulations show a good degree of agreement with satellite photographs and descriptions of the spill, showing that those models represent a valuable tool for decision-makers and emergency planners.

## Bibliographical References


1. Benqué, J. P., Hauguel, A. & Viollet, P. L. in *Engineering Applications of Computational Hydraulics II*, Pitman Advanced Publ. Programme, 1980.

2. Brooks, A. & Hughes, T. J. R. Streamline Upwind/Petrov-Galerkin Formulations for Convection Dominated Flows with Particular Emphasis on the Incompressible Navier-Stokes Equations, *Computer Methods in Applied Mathematics and Engineering*, 1982, **32**, 199-259.

3. Carbonel, C. Modelling of Coastal Ocean Response Induced by Wind Fields, *Laboratório Nacional de Computação Científica*, 1995.

4. Cuesta, I., Grau, F. X. & Giralt, F. Numerical Simulation of Oil Spills in a Generalized Domain, *Oil & Chemical Pollution*, 1990, **7**, 143-159.

5. Fay, J. A. The Spread of Oil Slicks on a Calm Sea. *Oil in the Sea*, 1969, Plenum Press, 53-63.

6. Meyer, J. F. C. A. Derrames de Petróleo em Águas Costeiras: Modelagem Matemática e Simulação Numérica, *in III° Simpósio de Ecossistemas da Costa Brasileira*, pp. 238-247, Serra Negra, São Paulo, Brasil, 1993.

7. Meyer, J. F. C. A., Cantão, R. F. & Poffo, I. R. F.: (1998), Oil Spill Movement in Coastal Seas: Modelling and Numerical Simulations, in Brebbia, C.A. (ed.), Oil Spill 98, Comp. Mech. Publ., Southampton, 1998, pp. 76-87.

8. Stolzenbach, K. D., Madsen, O. S., Adams, E. E., Pollack, A. M. & Cooper, C.K. A Review and Evaluation of Basic Techniques for Predicting the Behaviour of Surface Oil Slicks. M.I.T. *Sea Grant Report*, 1977, **222**, Mass. Inst. of Technol.

9. Valencia, M.J.: East Asian Seas: Hypothetical Oil Spill Trajectories. *Mar. Pollut. Bull.*, **23**, 1991.