Short Term Hydroelectric Scheduling Combining Network Flow and Interior Point Approaches

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Abstract

In this paper, short term hydroelectric scheduling is formulated as a network flow optimization model and solved by interior point methods. The primal-dual and predictor-corrector versions of such interior point methods are developed and the resulting matrix structure is explored. This structure leads to very fast iterations since it avoids computation and factorization of impedance matrices. For each time interval, the linear algebra reduces to the solution of two linear systems, either to the number of buses or to the number of independent loops. Either matrix is invariant and can be factored off-line. As a consequence of such matrix manipulations, a linear system which changes at each iteration has to be solved, although its size is reduced to the number of generating units and is not a function of time intervals. These methods were applied to IEEE and Brazilian power systems, and numerical results were obtained using a MATLAB implementation. Both interior point methods proved to be robust and achieved fast convergence for all instances tested.

Key words: Short term hydroelectric scheduling, Hydroelectric systems, Interior point methods, Network flow model.

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1 Introduction

Short term hydrothermal scheduling (STHS) is concerned with the commitment of and dispatch from hydro and thermal generating units, throughout a day usually on an hourly basis. STHS aims to optimize certain performance criteria associated with generation and/or transmission systems yet satisfy unit operational constraints and hydro generation targets established by long- and mid-term scheduling models. There is a vast bibliography describing different formulations and solution methodologies applied to STHS problem [1], [2], [3], [4], [5], [6], [7], [8], [9]. The usual decomposition suggested by many of these approaches consists of splitting the overall problem into two subproblems. The first of these is concerned with the commitment of the generating units, establishing which should be on or off at each hour of the day leading to integer programming models [2]. The second subproblem involves the computation of the optimal generation levels for hydro and thermal units subject to operational constraints on generation and transmission given the unit commitment established by the first subproblem. Solving the second subproblem efficiently is crucial, since it allows the solution of the first subproblem for large scale real systems. A large number of approaches have been proposed to deal with this second subproblem [10], [11], [12], [13], [14], [15], [16] and it is the subject of the present paper.

Most of the early formulations proposed for the STHS problem ignore the transmission system, or represent it in an excessively simplified way (such as the representation of a whole network as a single bus). These approaches do not lead to satisfactory results [11], [17]. In more recent works, the transmission system is represented in a more accurate way, requiring higher computational effort and generally adopting some sort of decomposition approach [10], [11], [13], [14], [16].

The STHS model presented in this paper is concerned with systems which are predominantly formed of hydro generators, such as that found in Brazil (92% hydro generation). In such systems, thermal units can be dispatched as flat as possible a priori, since the fluctuation due to differential demand can be dealt with hydro units. In this case, the costs of start up and shut down of generating units, as well as, the constraints associated with minimum up and down time and ramp rates are not as demanding as those involved in predominant thermal systems [18], [19]. Moreover, the usual objective of minimizing thermal generating costs can be replaced by the minimization of hydro generation losses associated with variations in tailrace elevation, penstock head losses, and turbine-generator efficiency [14]. Furthermore, the representation of water balance equations including water time delays and bounds on storage and discharge need not be taken into account, since the influence of a specific hydro unit on the operation of its immediately downstream neighbor is generally negligible for a planning horizon of a single day. Indeed, if the hydro generation targets are properly established, the variations in forebay elevation due to different scheduling have not effect on the solution. The solution provided by the STHS model proposed here should be hydraulically simulated in order to verify its feasibility.

In the approach proposed here, the transmission system is included in the STHS problem as a network flow model in which both Kirchhoff node and loop laws are considered. A network solution such as this is equivalent to that provided by a DC optimal load flow model [14], but the present approach differs from previous proposals in the sense that it requires no decomposition to solve the problem, since both primal-dual and predictor-corrector interior point (IP) methods are directly applied to the general STHS problem. A similar approach was adopted for an economic dispatch problem [15].

The network flow model adopted to represent the transmission system handles network flows explicitly [20], [21], allowing direct access both to capacity constraints and to transmission system losses. This feature also allows the representation of line control devices such as FACTS, as is discussed in [22], [23]. Interior point methods have been successfully applied to various problems involving the optimization of power systems, including short term scheduling [16], economic dispatch [15], unit commitment [24], state estimation [25], optimal power flow [26], [27] and reactive power dispatch [28], [29], [30]. Reviews of some of the applications of interior point methods in power systems are given in [31] and [32]. These studies have shown that IP methods have certain features that are very attractive for large scale optimization problems, such as robustness and efficiency. The application of IP methods for the generation scheduling problem discussed here shows that the same advantages accrue.

The generation scheduling approach proposed here thus aggregates the modeling abilities of network flow models with the efficiency and robustness of IP methods without requiring decomposition during the process of solution. It thus takes full advantage of the special structure of the problem by reducing the linear systems to be solved to a single dimension, either the number of buses, or the number of independent loops. Both matrices are invariant for all iterations and can be factored off-line. Numerical results involving IEEE test systems and the Brazilian power system are reported. A sensitivity study was performed using the IEEE30 test system to highlight the nature of the solution and the influence of the various parameters. The results showed that in all tests the IP methods converge after few iterations, thus indicating a robust and efficient tool for short term scheduling.

The paper is organized as follows: Section II describes the network flow approach for short term scheduling problem. Section III presents both primal-dual and predictor-corrector versions of IP methods developed to solve the problem. Implementation details and matrix structure manipulations are highlighted in Section IV, while Section V presents the numerical results and Section VI presents the conclusions.

2 Short Term Hydroelectric Scheduling Model

The formulation adopted here is designed to provide optimal utilization of the hydro units resources available. This formulation was proposed in [14] and is stated as follows:

$$\begin{aligned}
\min \quad & \sum_{t=1}^{T} \phi_t(f_t, p_t) && (1)\\
\text{subject to} \quad & Af_t = p_t - l_t, \quad Xf_t = 0 && (2)\\
& f^{min} \le f_t \le f^{max}, p^{min} \le p_t \le p^{max} \qquad \forall t \in (1, \dots, T) && (3)\\
& \sum_{t=1}^{T} p_t = q && (4)
\end{aligned}$$

where: $\sum_{t=1}^{T} \phi_t(f_t, p_t)$ is the objective function to be minimized; f_t represents the active power flow variables; p_t represents the active power generation variables; A represents the network incidence matrix; X represents the network loop reactance matrix; l_t represents the active load vector; f^{max}, f^{min}, p^{max} and p^{min} are bounds for the active power flow and generation variables; q represents the generating unit targets; and T defines the total number of intervals.

The objective function used in the model is a weighted combination of two separable quadratic functions

$$\phi_t(f_t, p_t) = \alpha_t \phi_l(f_t) + \beta_t \phi_g(p_t) \qquad (5)$$

where α_t and β_t are weight constants. This is a general function in the sense that it can represent most of the objectives traditionally used in the literature to dispatch active power. The quadratic function $\phi_l(f_t)$ can be associated with active power losses in the transmission system, whereas the function $\phi_g(p_t)$ can represent hydro generation loss [14]. These functions are generally written as follows:

$$\phi_l(f_t) = \frac{1}{2} f_t{}' R f_t, \qquad \phi_g(p_t) = \frac{1}{2} p_t{}' Q p_t + c_p' p_t$$

where, R is the diagonal resistance matrix, and Q and c_p are the diagonal matrix of quadratic coefficients and the coefficient vector of the linear term for generation loss, respectively.

The transmission system is represented by a network flow model (2) which considers both Kirchhoff laws (both node and branch laws). Equation (3) represents the bounds for active power flow and generation and (4) represents the generating target for the hydroelectric units established by long- or mid-term scheduling models.

3 Solution Technique

In order to simplify the notation, let us assume that the lower bounds in (3) are all zero and that $\alpha_t = \beta_t = 1$ in (5).

The dual problem for the short term schedule model (1)–(4) is given by [33]:

$$\begin{array}{ll} \text{maximize} & \sum_{t=1}^{T} (l_t' y_t - (f^{max})' w_t^f - (p^{max})' w_t^p - \frac{1}{2} f_t' R f_t - \frac{1}{2} p_t' Q p_t) \\ \text{subject to} & B' y_t + z_t^f - w_t^f - R f_t = c_f \\ & y_a - y(p_t) + z_t^p - w_t^p - Q p_t = c_p \qquad \forall t \in (1, \ldots, T) \\ & (z_t^p, w_t^p) \geq 0, (z_t^f, w_t^f) \geq 0 \end{array} \tag{6}$$

where $B = \begin{pmatrix} A \\ X \end{pmatrix}$; $d = \begin{pmatrix} l \\ 0 \end{pmatrix}$; z_t^f and z_t^p are slack variables; $y(p_t)$ are the entries on the dual vector y_t associated with generation buses at time interval t; and y_a represents the dual variables related to additional constraints.

The optimality conditions for primal and dual problems are given by primal and dual feasibility and the following *complementarity conditions*:

$$\begin{cases} F_t z_t^f = 0, \quad P_t z_t^p = 0 \\ S_t^f w_t^f = 0, \; S_t^p w_t^p = 0 \; \forall t \in (1, \ldots, T) \end{cases}$$

where, s_t^p and s_t^f are slack variables for the bound constraints on active power generation and transmission, respectively. Moreover, the notation $F = diag(f)$ for diagonal matrices formed by vectors is introduced.

The majority of primal-dual interior point methods can be seen to be variants of the Newton method applied to optimality conditions. The following outlines a framework for such methods, adopting $x = \cup_{i=1}^{t}(f_t, p_t, s_t^f, s_t^p)$, $v = \cup_{i=1}^{t}(z_t^f, z_t^p, w_t^f, w_t^p)$ and $u = (y_a, \cup_{i=1}^{t}(y_t))$.

Method 3.1 (Interior Point Methods) *Given* u^0 *and* $(x^0, v^0) > 0$.
For $k = 0, 1, 2, \ldots$, *do*

(1) *Choose* $\sigma^k \in [0, 1)$ *and set* $\mu^k = \sigma^k(\frac{\gamma^k}{n})$ *where,* $\gamma^k = (x^k)'v^k$ *and* n *is the size of* x.
(2) *Compute the Newton search directions* $(\Delta x^k, \Delta u^k, \Delta v^k)$.
(3) *Choose an appropriate step length to remain interior* $\alpha^k = \min(1, \tau^k \rho_p^k, \tau^k \rho_d^k)$ *where* $\tau^k \in (0, 1)$, $\rho_p^k = \frac{-1}{\min_i\left(\frac{\Delta x_i^k}{x_i^k}\right)}$ *and* $\rho_d^k = \frac{-1}{\min_i\left(\frac{\Delta v_i^k}{v_i^k}\right)}$.
(4) *Form the new iterate* $(x^{k+1}, u^{k+1}, v^{k+1}) = (x^k, u^k, v^k) + \alpha^k(\Delta x^k, \Delta u^k, \Delta v^k)$.

The step length for both primal and dual variables is the same, because for quadratic problems primal variables appear in the dual problem constraint set. The parameters σ and τ and the starting point are discussed later. Newton search directions are defined by the following linear system [2]:

$$\begin{cases} A\Delta f_t - \Delta p_t = r(n_t) \\ X\Delta f_t = r(v_t) \\ \Delta f_t + \Delta s_t^f = r(f_t) \qquad \forall t \in (1, \ldots, T) \\ \Delta p_t + \Delta s_t^p = r(p_t) \\ B'\Delta y_t + \Delta z_t^f - \Delta w_t^f - R\Delta f_t = r(y_t) \\ \Delta y_a - \Delta y(p_t) + \Delta z_t^p - \Delta w_t^p - Q\Delta p_t = r(g_t) \end{cases} \tag{7}$$

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[2] From now on, the superscript k will be omitted to provide a cleaner notation.

$$\begin{cases} Z_t^f \Delta f_t + F_t \Delta z_t^f = r(z_t^f) & \\ Z_t^p \Delta p_t + P_t \Delta z_t^p = r(z_t^p) & \forall t \in (1, \ldots, T) \\ W_t^f \Delta s_t^f + S_t^f \Delta w_t^f = r(w_t^f) & \\ W_t^p \Delta s_t^p + S_t^p \Delta w_t^p = r(w_t^p) & \end{cases} \tag{8}$$

and by

$$\sum_{t=1}^{T} \Delta p_t = r(m) \equiv q - \sum_{t=1}^{T} p_t \tag{9}$$

where the residuals are given by

$$\begin{cases} r(n_t) = l_t^v - Af_t + p_t & \\ r(v_t) = l^x - Xf_t & \\ r(f_t) = f^{max} - f_t - s_t^f & \\ r(p_t) = p^{max} - p_t - s_t^p & \\ r(y_t) = c_f - B'y_t - z_t^f + w_t^f + Rf_t & \\ r(g_t) = c_p - y_a + y(p_t) - z_t^p + w_t^p + Qp_t & \forall t \in (1, \ldots, T) \\ r(z_t^f) = \mu e - F_t Z_t^f e & \\ r(z_t^p) = \mu e - P_t Z_t^p e & \\ r(w_t^f) = \mu e - S_t^f W_t^f e & \\ r(w_t^p) = \mu e - S_t^p W_t^p e. & \end{cases}$$

and e is the column vector composed exclusively of ones.

3.2 The Predictor-Corrector Method

For the predictor-corrector approach [34], two linear systems have to be solved. First the *affine directions* $(\Delta\tilde{x}, \Delta\tilde{u}, \Delta\tilde{v})$ are computed by solving (7 to 9) for $\mu = 0$. The search directions are then given by solving (7), (9) and the following:

$$\begin{cases} Z_t^f \Delta f_t + F_t \Delta z_t^f = \tilde{r}(z_t^f) & \\ Z_t^p \Delta p_t + P_t \Delta z_t^p = \tilde{r}(z_t^p) & \forall t \in (1, \ldots, T) \\ W_t^f \Delta s_t^f + S_t^f \Delta w_t^f = \tilde{r}(w_t^f) & \\ W_t^p \Delta s_t^p + S_t^p \Delta w_t^p = \tilde{r}(w_t^p) & \end{cases}$$

where

$$\begin{cases} \tilde{r}(z_t^f) = \mu e - F_t Z_t^f e - \Delta \tilde{F}_t \Delta \tilde{Z}_t^f e \\ \tilde{r}(z_t^p) = \mu e - P_t Z_t^p e - \Delta \tilde{P}_t \Delta \tilde{Z}_t^p e \\ \tilde{r}(w_t^f) = \mu e - S_t^f W_t^f e - \Delta \tilde{S}_t^f \Delta \tilde{W}_t^f e \\ \tilde{r}(w_t^p) = \mu e - S_t^p W_t^p e - \Delta \tilde{S}_t^p \Delta \tilde{W}_t^p e \end{cases}$$

3.3 Implementation Issues

The parameters τ and σ have fixed values: $\tau = 0,99995$ and $\sigma = n^{-\frac{1}{2}}$. For the predictor-corrector approach, the perturbation is given by the following term: $(\frac{\tilde{\gamma}}{\gamma})^2(\frac{\tilde{\gamma}}{n^2})$, where $\tilde{\gamma} = (x + \Delta\tilde{x})'(v + \Delta\tilde{v})$ although if $\gamma < 1$, then $\mu = (\frac{\gamma}{n})^2$. The starting point is suggested to be the following: $y_a = 0$,

$$\begin{array}{ll} f_t = s_t^f = \frac{f^{max}}{2} & \\ p_t = s_t^p = \frac{p^{max}}{2} & \\ y_t = 0 & \forall t \in (1, \ldots, T) \\ z_t^f = w_t^f = (R + I)e & \\ z_t^p = w_t^p = e. & \end{array} \tag{10}$$

Another starting point can be obtained from the static model [35]. For each time interval, an optimal power flow problem is solved for a relaxed tolerance, and this solution is used as the starting point for the respective time interval. In such an approach, only the target constraints are not satisfied. The starting point (10) is used for the first time interval, with the solution of each interval used to start the next.

4 Linear System Solution

Since both linear systems share the same matrix, the following discussion will consider only the system involving (7), (8) and (9). The dimension of this linear system can be reduced by elimination involving various sets of variables without changing the sparse pattern of the matrix. First, slack variables are eliminated:

$$\begin{aligned} \Delta z_t^f &= (F_t)^{-1}(r(z_t^f) - Z_t^f \Delta f_t) \\ \Delta z_t^p &= (P_t)^{-1}(r(z_t^p) - Z_t^p \Delta p_t) \\ \Delta w_t^f &= (S_t^f)^{-1}(r(w_t^f) - W_t^f \Delta s_t^f) \qquad \forall t \in (1, \dots, T) \\ \Delta w_t^p &= (S_t^p)^{-1}(r(w_t^p) - W_t^p \Delta s_t^p) \\ \Delta s_t^f &= r(s_t^f) - \Delta f_t \\ \Delta s_t^p &= r(s_t^p) - \Delta p_t. \end{aligned}$$

reducing it to

$$\begin{cases} A\Delta f_t - \Delta p_t = r(n_t) \\ X\Delta f_t = r(v_t) \\ B'\Delta y_t - D_t^f \Delta f_t = r(a_t) \qquad \forall t \in (1, \dots, T) \\ \Delta y_a - \Delta y_t^p - D_t^p \Delta p_t = r(b_t) \end{cases} \tag{11}$$

where $D_t^f = (F_t)^{-1} Z_t^f + (S_t^f)^{-1} W_t^f + R$, $D_t^p = (P_t)^{-1} Z_t^p + (S_t^p)^{-1} W_t^p + Q$,
$r(a_t) = r(y_t) - F_t^{-1} r(z_t^f) + (S_t^f)^{-1} r(w_t^f) - (S_t^f)^{-1} W_t^f r(f_t)$ and
$r(b_t) = r(g_t) - P_t^{-1} r(z_t^p) + (S_t^p)^{-1} r(w_t^p) - (S_t^p)^{-1} W_t^p r(p_t)$.
note that only inverse diagonal matrices are involved. Now the active power generation and transmission variables in (11) can be eliminated:

$$\begin{aligned} \Delta f_t &= -(D_t^f)^{-1}(r(a_t) - B'\Delta y_t) \qquad \forall t \in (1, \dots, T) \\ \Delta p_t &= -(D_t^p)^{-1}(r(b_t) + \Delta y(p_t) - \Delta y_a) \end{aligned}$$

giving rise to

$$(B(D_t^f)^{-1}B' + D_t)\Delta y_t - (D_t^p)^{-1}\Delta y_a = r_t \qquad \forall t \in (1, \dots, T) \tag{12}$$

$$\sum_{t=1}^{T} (D_t^p)^{-1}(\Delta y_a - \Delta y(p_t)) = r(s) \tag{13}$$

where $r_t = r(l_t) + B(D_t^f)^{-1}r(a_t) - D_t^p r(b_t)$, $r(l_t) = \begin{pmatrix} r(n_t) \\ r(v_t) \end{pmatrix}$,

$r(s) = r(m) + \sum_{t=1}^{T}(D_t^p)^{-1}r(b_t)$ and D_t is a diagonal matrix with nonzero entries corresponding to the generation buses given by $(D_t^p)^{-1}$. Again, only inverse diagonal matrices are involved. Now, defining

$$(B(D_t^f)^{-1}B' + D_t)\Delta\hat{y}_t = r_t \tag{14}$$

and replacing Δy_t in (13), we obtain:

$$\sum_{t=1}^{T}(D_t^p)^{-1}(\Delta y_a - E_j(B(D_t^f)^{-1}B' + D_t)^{-1}E_j'(D_t^p)^{-1}\Delta y_a) = \tilde{r} \tag{15}$$

where matrix E_j is formed by the canonical vectors corresponding to the diagonal nonzero entries of $\tilde{D_t^p}$ and $\tilde{r} = r(s) + \sum_{t=1}^{T}\Delta\hat{y}_t$. Given Δy_a, the search directions Δy_t can be computed from (12).

4.1 Exploiting the Sparse Pattern of the Matrix

The fact that adding any canonical column e_j to B gives a square nonsingular matrix is crucial. This feature can be used to reduce the computational work per iteration of the interior point methods. Let us consider the matrix $\tilde{B} = [B\ e_j]$. The linear system in (14) can be rewritten as follows:

$$(\tilde{B}(\tilde{D}_t^f)^{-1}\tilde{B}' + \tilde{D}_t)\Delta y_t = r_t \qquad \forall t \in (1, \ldots, T) \tag{16}$$

where $(\tilde{D}_t^f)^{-1}$ contains an extra diagonal entry (jj) taken from D in order to form $\tilde{D}$. The efficient solution of (16) is presented in [35] using the Sherman-Morrison-Woodbury formula. It should be noted that the size of the system can be reduced to either the number of buses or the number of independent loops. Furthermore, the matrix of such a system is fixed for a given network and can be factored off-line. An off-line heuristic for computing a reactive matrix is also given in [35].

4.2 Computing Δy_a

Solving the linear system (15) would be very expensive, since the matrices $M_t + D_t = B(D_t^f)^{-1}B' + D_t$ have the dimension of transmission lines, while

Δy_a has the size of the number of generating units. The dimension of the linear system, however, can be reduced.

Applying the Sherman-Morrison-Woodbury formula to $M+D$[3] yields the following:

$$(M+E_{j-1}\tilde{D}_pE'_{j-1})^{-1}=M^{-1}-M^{-1}E_{j-1}\tilde{D}_p^{\frac{1}{2}}(I+\tilde{D}_p^{\frac{1}{2}}E'_{j-1}M^{-1}E_{j-1}\tilde{D}_p^{\frac{1}{2}})^{-1}\tilde{D}_p^{\frac{1}{2}}E'_{j-1}M^{-1}.$$

Using the relation

$$\begin{aligned}E'_{j-1}M^{-1}E_{j-1}&=E'_{j-1}(\tilde{B}\tilde{D}_f\tilde{B}')^{-1}E_{j-1}\\&=E'_{j-1}\tilde{B}^{-t}\tilde{D}_f^{-1}\tilde{B}^{-1}E_{j-1}\\&=W'_{j-1}\tilde{D}_f^{-1}W_{j-1}\\&=Z_{j-1}\end{aligned}$$

gives

$$(M+E_{j-1}\tilde{D}_pE'_{j-1})^{-1}=M^{-1}-M^{-1}E_{j-1}(\tilde{D}_p^{-1}+W'_{j-1}\tilde{D}_f^{-1}W_{j-1})^{-1}E'_{j-1}M^{-1}$$

hence,

$$\begin{aligned}E'_j(M+E_{j-1}\tilde{D}_pE'_{j-1})^{-1}E_j&=E'_jM^{-1}E_j-E'_jM^{-1}E_{j-1}(\tilde{D}_p^{-1}+Z_{j-1})^{-1}E'_{j-1}M^{-1}E_j\\&=Z_j-Z_jE_{j-1}(\tilde{D}_p^{-1}+Z_{j-1})^{-1}E'_{j-1}Z_j\\&=(Z_j^{-1}+E_{j-1}\tilde{D}_pE'_{j-1})^{-1}\end{aligned}$$

where the Sherman-Morrison-Woodbury formula was once again applied. Defining

$$N=\tilde{D}_p^{-1}(Z_j^{-1}+E_{j-1}\tilde{D}_pE'_{j-1})^{-1}\tilde{D}_p^{-1},$$

Equation (15) is thus reduced to

$$\sum_{t=1}^{T}((D_t^p)^{-1}-N_t)\Delta y_a=\tilde{r} \qquad (17)$$

whose matrix has the size of generating units.

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[3] In this deduction the subscript t is dropped for a cleaner notation.

Table 1
Load Factor

Hour	Factor	Hour	Factor	Hour	Factor
1:00	0.7948	2:00	0.7425	3:00	0.7255
4:00	0.7222	5:00	0.7345	6:00	0.7816
7:00	0.9012	8:00	0.9832	9:00	1.0535
10:00	1.0896	11:00	1.0976	12:00	1.0888
13:00	1.0557	14:00	1.0823	15:00	1.0814
16:00	1.0846	17:00	1.1134	18:00	1.1714
19:00	1.2998	20:00	1.2393	21:00	1.1658
22:00	1.1089	23:00	1.0000	24:00	0.8828

5 Numerical Results

All the experiments were carried out on a Sun Ultra-60 station using the IEEE standard for floating point arithmetic double precision with tolerance set to 10^{-6}. The primal-dual and predictor-corrector methods were implemented in MATLAB.

The main goal of the experiments was to show the advantages of these methods in relation to both speed and robustness. In all experiments, $f^{min} = -f^{max}$ was used for active power flow, and $p^{min} = 0$MW was used for active power generation. In order to simplify the interpretation of the results, only pure quadratic cost functions for generation were used, i. e., $c_p = 0$, and the same quadratic coefficients were used for all units.

The load for each time interval is given by the basic load of the system under consideration multiplied by the weight under "Factor" in Table 1. These factors mimic the load curve of the Brazilian company CESP for a typical week day.

The starting point for the primal variables are given by Equation (10). The best starting point for the dual variables is found by the solution of the static problems, with the tolerance set to 10^{-3}. The starting point used is critical, since it helps reduce the number of iterations.

Figures 1–4 show the results for the IEEE30 test system using the predictor-corrector method. Limits on power generation and flow were initially fixed to be large enough to obtain optimal solutions without active bounds. Four different cases were considered.

Figure 1, shows power generation scheduling when generation costs are ignored and only transmission loss is taken into account ($\alpha_t = 1$ and $\beta_t = 0$). Furthermore, no generation targets were enforced for the generating units in this case. Therefore, this output corresponds to the best solution from the transmission point of view at each hour of the day. Note that Unit 13, the most expensive in terms of transmission loss, shows a low and flat generation scheduling profile as opposite to Unit 2, the less expensive one, which shows a generation scheduling profile quite similar to the load curve. The remaining units have shown an intermediate behavior.

In Fig. 2, a generation target is enforced for each unit; for example, a large target is assigned to Unit 13 and a small one to Unit 2. The results show a vertical shift in unit scheduling in order to accommodate these generation targets without changing the previous profiles since the overall objective is still to minimize transmission loss.

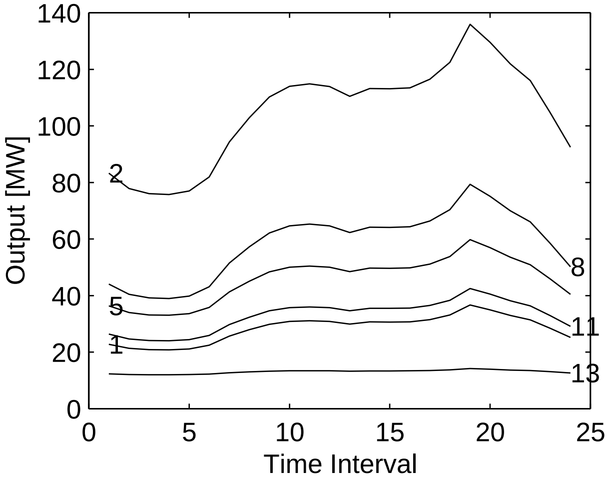

Fig. 1. Transmission loss only – ignoring targets

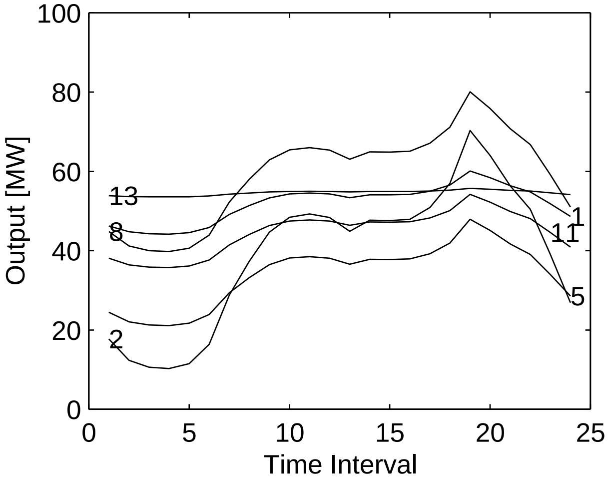

Fig. 2. Transmission loss only – including targets

In Fig. 3, transmission loss is ignored and only the minimization of generation costs is considered ($\alpha_t = 0$ and $\beta_t = 1$). As in the previous case, generation targets are maintained. This new solution however, results in drastic changes providing identical output profiles for all generators, since all generators have the same generation costs, the load curves for all buses are identical, and there is no active bounds on power generation and flow.

In Fig. 4, the two objectives are combined using the following weights: ($\alpha_t = mc_t$ and $\beta_t = 1$), where mc_t is the marginal cost of the generating units at time interval t in the previous solution ($\alpha_t = 0$ and $\beta_t = 1$). The determination of these weights was designed to compute the cost, in monetary units (\$), of generating the transmission loss (MW) so that both terms in the objective function could be expressed in the same units. As can be seen here, the solution is similar to that obtained when considering only generation costs; this suggests that they are more critical than transmission costs. Indeed, these

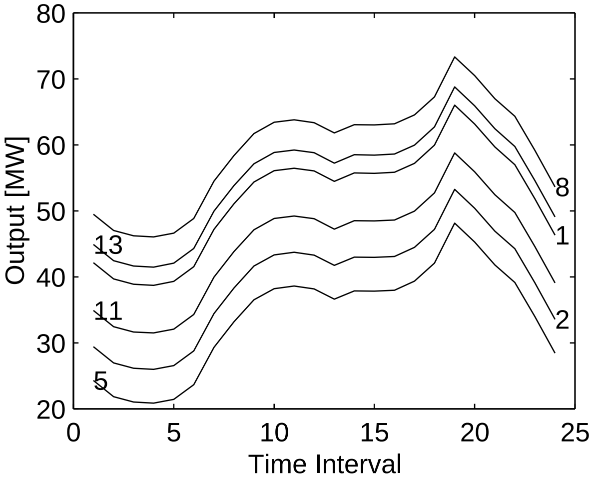

Fig. 3. Generation costs only – including targets

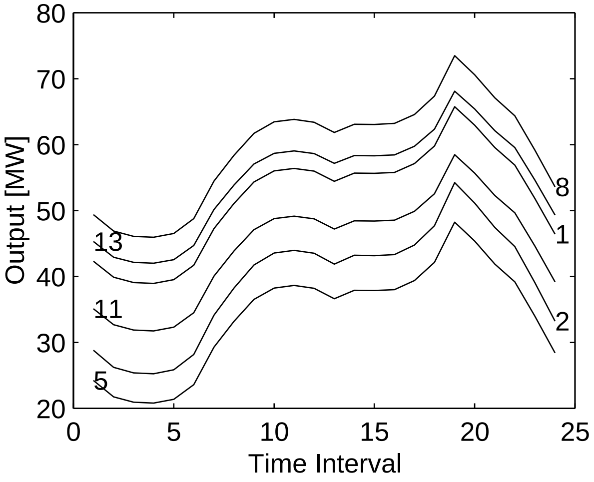

Fig. 4. Combined Generation and Transmission costs

results are due to the relatively low values of transmission loss with respect to total system load.

For the remaining experiments, the combined objective function ($\alpha_t = mc_t$ and $\beta_t = 1$) was adopted, and a series of successive modifications of system parameters was introduced.

In Fig. 5, the costs for Unit 13 are doubled. This leads to a more uniform profile of the unit accompanied by compensation in the scheduling of all the other units, where profile become slightly more variable.

In Fig. 6, the upper bound for Unit 8 was set to 65MW in order to simulate active bounds on power generation. In this case, the solution required 14 iterations using the primal-dual method and the predictor-corrector was used for the remaining iteration, thus leading to a more robust approach. The primal-

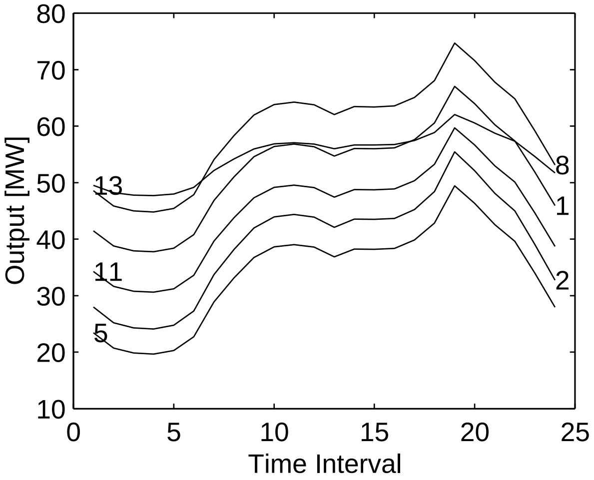

Fig. 5. Cost of Unit 13 doubled

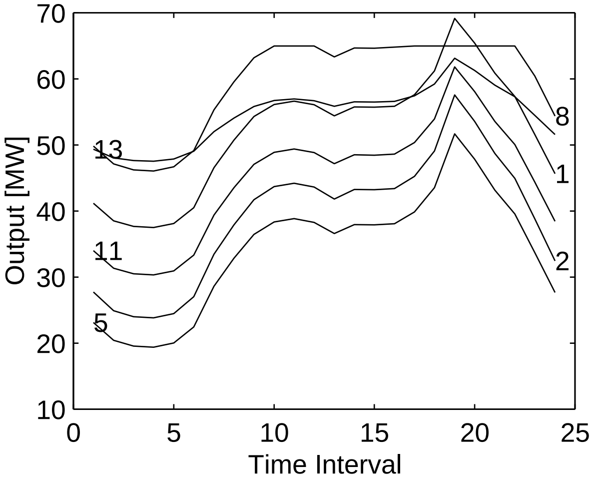

Fig. 6. Unit 8 bounded

dual method seems to priorize optimality while the predictor-corrector variant priorizes factibility.

In Fig. 7, the capacity for line 2–5 was set to 60MW, simulating active bounds in the transmission system. The behaviour of both methods was found to be similar to what was found in the previous experiment; the convergence was achieved in 19 iterations. Fig. 8 shows the flow along line 2–5 with and without the 60MW capacity.

Table 2 presents the number of iterations and running time for the experiments shown in Figures 1 to 7. The consideration of generation costs in the objective function (Cases 3 and 4), significantly reduces the number of iterations required to achieve convergence. Computational time for Cases 6 and 7 increased more slowly than did the number of iterations since the primal-dual method has a less expensive iteration than the predictor-corrector method.

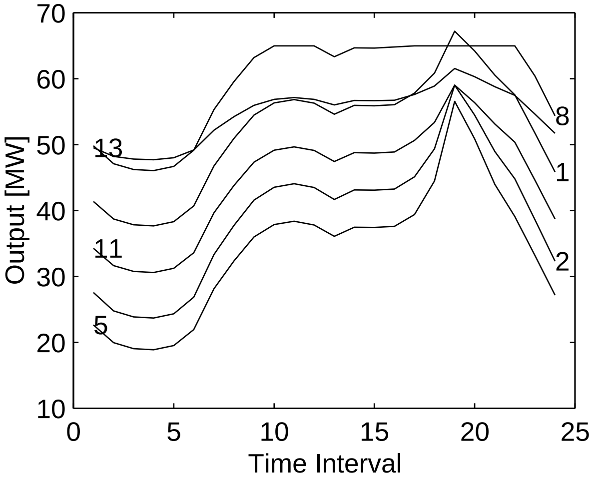

Fig. 7. Line 2–5 Bounded

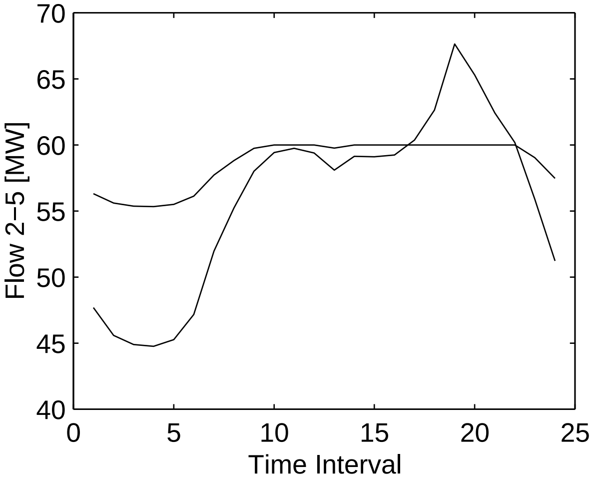

Fig. 8. Flow in Line 2–5

Finally, Table 3 shows the number of iterations and the running time for several large scale systems using the predictor-corrector method. Systems SSE 1654 and SSE 1732 are two representations of the South – Southeast Brazilian power system, whereas Brazil 1993 represents the entire interconnected Brazilian power system. These results show that the number of iterations continues small while the processing time increases slowly with the number of buses.

6 Conclusions and Future Work

Short term hydrothermal scheduling has been formulated in this paper as a network flow optimization model; the problem was solved by interior point methods. One advantage of the network flow modeling approach is that transmission capacities and loss are neatly handled, since power flows are explicitly represented. Moreover, the specific structure of the model is suitable for applying interior point methods leading to very fast iterations, since most of the computational work can be performed off-line.

There are two main advantages of these methods. The first is that they tend to be rapid due to the limited number of iterations necessary, moreover, these iterations are very fast. Thus, less than forty minutes are required to solve large

Table 2
IEEE30 – Iteration Count and CPU Time (sec.)

Case	Iterations	Time
1	6	3.21
2	6	3.23
3	3	1.90
4	2	1.25
5	2	1.37
6	15	4.79
7	19	6.51

Table 3
Iteration Count and CPU Time (seconds)

Problem	Buses	Iterations	Time
IEEE	118	3	16.5
SSE	1654	10	1987
SSE	1732	11	2072
Brazil	1993	10	2267

scale Brazilian systems using MATLAB. The second advantage involves robustness, since the methods obtain convergence without numerical instability even for loaded systems.

A case study with the IEEE30 system was also performed. Different objective functions and parameter settings were considered in order to highlight the problem features and their influence in the problem solution. As a consequence, it is possible to conclude that generation costs prevail over transmission costs and allow faster convergence. On the other hand, active bounds increase the number of iterations required for convergence, although the computational time increase more slowly since the primal-dual iterations are less expensive.

The present approach can easily be extended to hydrothermal scheduling once the unit commitment of thermal units is established. Thermal units have no generation targets, which simplifies the model somewhat. Ramp constraints, however, must be taken into consideration, although they pose no extra computational effort for the method presented here since such constraints are involved only for consecutive time intervals. The work per iteration is determined principally by the number of time intervals and hydroelectric units, whereas the extra overhead due to the presence of thermal units is insignificant.

Acknowledgments

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References

[1] H. Habibollahzadeh and J. A. Bubenko, "Application of decomposition techniques to short term operation planning of hydrothermal power system," *IEEE Transactions on Power Systems*, vol. PWRS-1, no. 1, pp. 41–47, 1986.

[2] G. B. Shebl and G. N. Fahd, "Unit commitment literature synopsis," *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 128–135, 1994.

[3] R. Baldick, "The generalized unit commitment problem," *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 465–475, 1995.

[4] C. Li, R. B. Johnson, and A. J. Svoboda, "A new unit commitment method," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 113–119, 1997.

[5] H. T Yang, P. C. Yang, and Huang C. L, "A parallel genetic algorithm approach to solving the unit commitment problem: implementation on the transputer networks," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 661–669, 1997.

[6] C. Li, R. B. Johnson, A. J. Svoboda, Tseng C., and E. Hsu, "A robust unit commitment algorithm for hydro-thermal optimization," *IEEE Transactions on Power Systems*, vol. 13, no. 3, pp. 1051–1056, 1998.

[7] M. S. Salam, Nor K. M., and Hamdan A. R., "Hydrothermal scheduling based lagrangian relaxation approach to hydrothermal coordination," *IEEE Transactions on Power Systems*, vol. 13, no. 1, pp. 226–235, 1998.

[8] H. Ma and S. M. Shahidehpour, "Unit commitment with transmission security and voltage constraints," *IEEE Transactions on Power Systems*, vol. 14, no. 2, pp. 757–764, 1999.

[9] N. Alguacil and Conejo A. J., "Multiperiod optimal power flow using benders decomposition," *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 196–201, 2000.

[10] T. Ohishi, S. Soares, and M. F. H. Carvalho, "A short term hydrothermal scheduling approach for dominantly hydro systems," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 637–643, 1991.

[11] J. Batut and A. Renaud, "Daily generation scheduling optimization with transmission constraints: a new class of algorithms," *IEEE Transactions on Power Systems*, vol. 7, no. 3, pp. 982–989, 1992.

[12] P. Franco, M. F. Carvalho, and S. Soares, "A network flow model for short-term hydro-dominated hydrothermal scheduling problem," *IEEE Transactions on Power Systems*, vol. 9, no. 2, pp. 1016–1021, 1994.

[13] X. Guan, P. B. Luh, and Zhang L., "Nonlinear approximation method in lagrangian relaxation-based algorithms for hydrothermal scheduling," *IEEE Transactions on Power Systems*, vol. 10, no. 2, pp. 772–778, 1995.

[14] S. Soares and C. T. Salmazo, "Minimum loss predispatch model for hydroelectric systems," *IEEE Transactions on Power Systems*, vol. 12, no. 3, pp. 1220–1228, 1997.

[15] G. Irissari, L. M. Kimball, K. A. Clements, A. Bagchi, and P. W. Davis, "Economic dispatch with network and ramping constraints via interior point methods," *IEEE Transactions on Power Systems*, vol. 13, no. 1, pp. 236–242, 1998.

[16] H. Wei, H. Sasaki, and J. Kubokawa, "A decoupled solution of hydro-thermal optimal power flow problem by means of interior point method and network flow programming," *IEEE Transactions on Power Systems*, vol. 13, no. 2, pp. 286–293, 1998.

[17] D. Chattopadhyay, "Daily generation scheduling: quest for new models," *IEEE Transactions on Power Systems*, vol. 13, no. 2, pp. 624–629, 1998.

[18] O. Nilsson and D. Sjelvgren, "Variable splitting applied to modeling of start-up in short term hydro generation scheduling," *IEEE Transactions on Power Systems*, vol. 12, no. 2, pp. 770–775, 1997.

[19] O. Nilsson and D. Sjelvgren, "Hydro unit start-up and their impact on short term scheduling strategies of swedish power producers," *IEEE Transactions on Power Systems*, vol. 12, no. 1, pp. 38–43, 1997.

[20] D. Sjelvgren and T. S. Dillon, "Seasonal planning of hydro-thermal systems using the network flow concept," *Proceedings Seventh Power Systems Computation Conference, Lausanne, Switzerland*, pp. 1191–1199, July 1981.

[21] T. S. Dillon, S. Anderson, and D. Sjelvgren, "Optimal operations planning in a large hydro-thermal power system," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-102, pp. 3644–3651, 1983.

[22] G. Shaoyun and T. S. Chung, "Optimal active power flow incorporating FACTS devices with power flow control constraints," *Electrical Power & Energy Systems*, vol. 20, no. 2, pp. 321–326, 1998.

[23] S. Y. Ge and T. S. Chung, "Optimal active power flow incorporating power flow control needs in flexible AC transmission systems," *IEEE Transactions on Power Systems*, vol. 14, pp. 738–744, 1999.

[24] M. Madrigal and V. H. Quintana, "An interior-point/cutting plane method to solve unit commitment problems," *Proc. of PICA 99*, pp. 203–209, 1999.

[25] J. M. Ramírez and E. E. Barocio, "Solving state estimation in power systems by an interior point method," *Electrical Power & Energy Systems*, vol. 22, pp. 355–365, 2000.

[26] X. Yan and V. H. Quintana, "An infeasible interior-point algorithm for optimal power-flow problems," *Electrical Power Systems Research*, vol. 39, pp. 39–46, 1996.

[27] A. Garzillo, M. Innorta, and R. Ricci, "The problem of the active and reactive optimum power dispatching solved by utilizing a primal-dual interior point method," *Electrical Power & Energy Systems*, vol. 20, no. 6, pp. 427–434, 1998.

[28] S. Granville, "Optimal reactive power dispatch through interior point methods," *IEEE Transactions on Power Systems*, vol. 9, no. 1, pp. 136–146, 1994.

[29] A. Garzillo, M. Innorta, and R. Ricci, "The flexibility of interior point based power flow algorithms facing critical network situations," *Electrical Power & Energy Systems*, vol. 21, pp. 579–584, 1999.

[30] E. Rezania and S. M. Shahidehpour, "Real power loss minimization using interior point methods," *Electrical Power & Energy Systems*, vol. 23, pp. 45–56, 2001.

[31] J. A. Momoh, M. E. El-Hawary, and R. Adapa, "A review of selected optimal power flow literature to 1993 II Newton, linear programming and interior point methods," *IEEE Transactions on Power Systems*, vol. 14, pp. 105–111, 1999.

[32] V. H. Quintana, G. L. Torres, and J. Medina-Palomo, “Interior point methods and their applications to power systems: A classification of publications and software codes,” *IEEE Transactions on Power Systems*, vol. 15, no. 1, pp. 170–176, 2000.

[33] D. G. Luenberger, *Linear and Nonlinear Programming*, Addison-Wesley, Reading, 1984.

[34] S. Mehrotra, “On the implementation of a primal-dual interior point method,” *SIAM Journal on Optimization*, vol. 2, no. 4, pp. 575–601, 1992.

[35] A. R. L. Oliveira, S. Soares, and L. Nepomuceno, “Optimal active power dispatch combining network flow and interior point approaches,” *IEEE Transactions on Power Systems*, vol. 18, no. 4, pp. 1235–1240, November 2003.