◇ **Exercise 1.1-1** Prove the following properties of the material derivative

(i) $\dfrac{D}{Dt}(f+g) = \dfrac{Df}{Dt} + \dfrac{Dg}{Dt}$,

(ii) $\dfrac{D}{Dt}(f \cdot g) = f\dfrac{Dg}{Dt} + g\dfrac{Df}{Dt}$ (Leibniz or product rule),

(iii) $\dfrac{D}{Dt}(h \circ g) = (h' \circ g)\dfrac{Dg}{Dt}$ (chain rule).

◇ **Exercise 1.2-1** Derive a formula akin to the transport theorem and Kelvin's circulation theorem for

$$\frac{d}{dt}\int_{S_t} \mathbf{v} \cdot \mathbf{n}\, dA,$$

where S_t is a *moving surface* and $\mathbf{v}$ is a vector field.

◇ **Exercise 1.2-2** ***Couette flow.*** Let Ω be the region between two concentric cylinders of radii R_1 and R_2, where $R_1 < R_2$. Define $\mathbf{v}$ in cylindrical coordinates by

$$v_r = 0, \qquad v_z = 0,$$

and

$$v_\theta = \frac{A}{r} + Br,$$

where

$$A = -\frac{R_1^2 R_2^2(\omega_2 - \omega_1)}{R_2^2 - R_2^1} \quad \text{and} \quad B = -\frac{R_1^2\,\omega_1 - R_2^2\,\omega_2}{R_2^2 - R_1^2}.$$

Show that

(i) $\mathbf{v}$ is a stationary solution of Euler's equations with $\rho = 1$;

(ii) $\boldsymbol{\omega} = \nabla \times \mathbf{v} = (0, 0, 2B)$;

(iii) the deformation tensor is

$$D = -\frac{A}{r^2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and discuss its physical meaning;

(iv) the angular velocity of the flow on the two cylinders is ω_1 and ω_2.

◇ **Exercise 1.3-1** Find a stationary viscous incompressible flow in a circular pipe with radius $a > 0$ and with pressure gradient ∇p.