24. State and prove an analogue of Theorem 8.33 for functions on $\mathbb{R}$. (In addition to the hypotheses that f be locally absolutely continuous and that $f' \in L^p$ for some $p > 1$, you will need some further conditions f and/or f' at infinity to make the argument work. Make them as mild as possible.)

29. Given $\{a_k\}_0^\infty \subset \mathbb{C}$, let $S_n = \sum_0^n a_k$ and $\sigma_m = (m+1)^{-1} \sum_0^m S_n$.

a. $\sigma_m = (m+1)^{-1} \sum_0^m (m+1-k)a_k$.
b. If $\lim_{n\to\infty} S_n = \sum_0^\infty a_k$ exists, then so does $\lim_{m\to\infty} \sigma_m$, and the two limits are equal.
c. The series $\sum_0^\infty (-1)^k$ diverges but is Abel and Cesàro summable to $\frac{1}{2}$.

30. If $f \in L^1(\mathbb{R}^n)$, f is continuous at 0, and $\widehat{f} \geq 0$, then $\widehat{f} \in L^1$. (Use Theorem 8.35c and Fatou's lemma.)

35. The purpose of this exercise is to show that the Fourier series of "most" continuous functions on $\mathbb{T}$ do not converge pointwise.
a. Define $\phi_m(f) = S_m f(0)$. Then $\phi \in C(\mathbb{T})^*$ and $\|\phi\| = \|D_m\|_1$.
b. The set of all $f \in C(\mathbb{T})$ such that the sequence $\{S_m f(0)\}$ converges is meager in $C(\mathbb{T})$. (Use Exercise 34 and the uniform boundedness principle.)
c. There exist $f \in C(\mathbb{T})$ (in fact, a residual set of such f's) such that $\{S_m f(x)\}$ diverges for every x in a dense subset of $\mathbb{T}$. (The result of (b) holds if the point 0 is replaced by any other point in $\mathbb{T}$. Apply Exercise 40 in §5.3.)

40. $L^1(\mathbb{R}^n)$ is vaguely dense in $M(\mathbb{R}^n)$. (If $\mu \in M(\mathbb{R}^n)$, consider $\phi_t * \mu$ where $\{\phi_t\}_{t>0}$ is an approximate identity.)