4. Suppose that U and V are open in $\mathbb{R}^n$ and $\Phi : V \to U$ is a C^∞ diffeomorphism. Explain how to define $F \circ \Phi \in \mathcal{D}'(U)$ for any $F \in \mathcal{D}'(V)$.

5. Suppose that f is continuously differentiable on $\mathbb{R}$ except at $x_1, \ldots, x_m$, where f has jump discontinuities, and that its pointwise derivative df/dx (defined except at the x_j's) is in $L^1_{\text{loc}}(\mathbb{R})$. Then the distribution derivative f' of f is given by $f' = (df/dx) + \sum_1^m [f(x_j+) - f(x_j-)]\tau_{x_j}\delta$.

17. Suppose that $F \in \mathcal{S}'$. Show that

a. $(\tau_y F)\widehat{} = e^{-2\pi i\xi\cdot y}\widehat{F}$, $\tau_\eta \widehat{F} = [e^{2\pi i\eta\cdot x}F]\widehat{}$.

b. $\partial^\alpha \widehat{F} = [(-2\pi i x)^\alpha F]\widehat{}$, $(\partial^\alpha F)\widehat{} = (2\pi i\xi)^\alpha \widehat{F}$.

c. $(F \circ T)\widehat{} = |\det T|^{-1}\widehat{F} \circ (T^*)^{-1}$ for $T \in GL(n, \mathbb{R})$.

d. $(F * \psi)\widehat{} = \widehat{\psi}\widehat{F}$ for $\psi \in \mathcal{S}$.

2.3.4. (a) Prove that the derivative of $\chi_{[a,b]}$ is $\delta_a - \delta_b$.

(b) Compute $\partial_j \chi_{B(0,1)}$ on $\mathbf{R}^2$.

(c) Compute the Fourier transforms of the locally integrable functions $\sin x$ and $\cos x$.

(d) Prove that the derivative of the distribution $\log|x| \in \mathscr{S}'(\mathbf{R})$ is the distribution

$$u(\varphi) = \lim_{\varepsilon \to 0} \int_{\varepsilon \le |x|} \varphi(x)\frac{dx}{x}.$$

2.3.10. Show that the functions e^{inx} and e^{-inx} converge to zero in $\mathscr{S}'$ and $\mathscr{D}'$ as $n \to \infty$. Conclude that multiplication of distributions is not a continuous operation even when it is defined. What is the limit of $\sqrt{n}(1 + n|x|^2)^{-1}$ in $\mathscr{D}'(\mathbf{R})$ as $n \to \infty$?

2.3.11. (*S. Bernstein*) Let f be a bounded function on $\mathbf{R}^n$ with $\widehat{f}$ supported in the ball $B(0,R)$. Prove that for all multi-indices α there exist constants $C_{\alpha,n}$ (depending only on α and on the dimension n) such that

$$\left\|\partial^\alpha f\right\|_{L^\infty} \le C_{\alpha,n} R^{|\alpha|} \left\|f\right\|_{L^\infty}.$$